A Theory of Structural Change, Home Production and Leisure*

Fenicia Cossu†
University of Cagliari

Alessio Moro‡
University of Cagliari

F. Javier Rodriguez-Roman §
University of Cagliari

Silvio Tunis ¶
University of Cagliari

July 2023

Abstract

Why do agents consume more services relative to goods as income grows? We present a theory of structural change assuming that a representative household satisfies final needs by means of two home-production functions in *time* and either *goods* or *services* from the market. When calibrating the model to U.S. data, roughly half of structural change is accounted for by technological change allowing services to display a larger time saving than goods in satisfying final needs. Also, even if preferences are homothetic, the calibrated model generates endogenous income effects, which account for the remaining structural change generated by the model.

JEL Classification: D13, D24, O47.

Keywords: Structural Transformation, Home-Production, Time Use.

---

*We thank William Addessi, Paco Buera, Cristiano Cantore, Wei Jiang, Joe Kaboski, Miguel Leon-Ledesma, Vincenzo Merella, Andrii Parkhomenko, Omar Rachedi, Diego Restuccia, Yongseok Shin, Gianluca Violante, Fabrizio Zilibotti, and participants at the IX Workshop on Structural Transformation and Macroeconomic Dynamics in Cagliari, the STEG Theme 2 Workshop 2022, and the 47th Simposio of the Spanish Economic Association in Valencia for their useful comments. Moro thanks the Region of Sardinia for financial support, research grant F73C22001420007.

†Contact: fenicia.cossu@unica.it

‡Contact: amoro@unica.it

§Contact: franciscojavier.rodr@unica.it

¶Contact: silvio.tunis@unica.it

1 Introduction

Why do economies consume more services relative to goods as they grow richer? A robust empirical observation of the process of development is that the share of services in GDP and in aggregate consumption increases with the level of income. Evidence for the U.S. and for many other advanced countries typically shows that over time such an increase is due both to a growth in the relative price and in the relative quantity of services with respect to the rest of the economy, which suggests that a non-homothetic component into preferences is needed to fit the data (Buera and Kaboski, 2009, Boppart, 2014, Comin, Lashkari, and Mestieri, 2021, Buera, Kaboski, Rogerson, and Vizcaino, 2022 and Alder, Boppart, and Muller, 2022). The interpretation of such non-homothetic component however, is somewhat left unaddressed in the literature. Some contributions employing Stone-Geary preferences interpret the non-homothetic component of services as a reduced form of home production, suggesting that the latter is the source of an income elasticity of services larger than one (Kongsamut, Rebelo, and Xie, 2001). However, Moro, Moslehi, and Tanaka (2017), using home-production data for the U.S. constructed by Bridgman (2016), find that even in a context in which home production is explicitly introduced in a model of structural change, an exogenous non-homothetic component in preferences is needed for the model to account for the observed structural transformation in the U.S., which suggests that a basic home production sector is not sufficient to account for the required income effects.

Our paper contributes to the literature focusing on the role of home production for structural transformation, with an important caveat. Typically, this class of models assumes that the household can produce goods and services at home (or a subset of them) or purchase them in the market. This consumption choice is then tightly linked to the allocation of time between market work and home work. The resulting theoretical framework is one in which market and home goods/services are to some extent complements/substitutes. In this paper, we take another stand and assume that goods and services do not provide direct utility to the agent but are instead used as intermediate inputs in the production of final consumption. To fix concepts, we think of final consumption as a set of *needs* that the household can satisfy in two ways: either combining time with market-purchased goods (i.e. cooking a burger at home) or using time with market-purchased services (i.e. by going to the closest Mcdonald's). In this view, goods and services purchased in the market are, respectively, combined with household time by means of two types of home-production functions to obtain "final consumption quantities" of two different needs entering the utility function.

The model encompasses differences in the production of final needs (i.e. utility) that have the potential to account for structural transformation and are absent in standard models. We highlight two main mechanisms that contribute to shaping structural change. First, home production activities using services and goods are allowed to display a different degree of substitutability with time. This implies that households with different wage levels, for which the opportunity cost of time is different, might optimally choose a different ratio of purchased goods and services. This is because households at higher wage levels want to reduce the more expensive time in home production activities. However, the extent to which they can do this in the two home activities is different due to the different degree of substitutability with time. In this light, the model endogenously generates a non-homothetic component even if all functions in the model are homothetic.

Second, the two home production activities might display a different pace of time-saving technological change. Consider the following example. Assume that there are only two types of final consumption entering the utility function, which are described by "burgers cooked at home" and "Big Macs obtained in the market". The two satisfy similar needs of the consumer and thus are highly substitutable, though they are not perfect substitutes. In terms of time needed to consume the Big Mac, in the 1950s this amounted to physically reaching and entering the closest Mcdonald's. In the 1970s, the first Drive-Thrus were added to the restaurants, which reduced the time needed to purchase the burger. In the 1990s, the first McDelivery services were introduced, which further slashed the time to purchase the Big Mac. In the 2000s delivery providers like Deliveroo and Just Eat reduced the time to obtain the Big Mac to a few seconds online. Each of these innovations contemporaneously increased the quantity of services (the first input) and reduced the time (the second input) needed to consume *the same* final consumption product. Cooking a burger at home instead, requires roughly the same amount of time nowadays as it did in the 1950s. In this view, the amount of time needed to satisfy a need through the home production employing services shrinks at a faster rate than the amount of time needed to consume the type of home production employing goods. Thus, even if the cost of the service in the market increases relative to the cost of the goods needed to cook the burger at home, the time saved by the consumer in the first case might more than compensate for such change, inducing the consumer to increase the quantity purchased of services relative to goods.

Formally, the model is as follows. There are two market firms, one producing services and the other goods by means of a firm specific production function that employs time as the only input. Each firm experiences TFP growth over time. There is a representative household owing one unit of time in each period, optimally split into market work, leisure, and home production. The household has a consumption index that aggregates two types of final consumption: c_g and c_s. To obtain amounts for such kinds of consumption she needs to purchase some quantities of goods and services in the market and combine them with time according to two different home-production functions. Finally, there is a time-saving technological change in each home production function. This increases the productivity of time as technology advances, allowing to produce the same amount of final consumption with a reduced amount of this input.

Despite its simplicity and use of only homothetic functions, the model has the potential to generate structural change as observed in time-series data. Three key elasticities and four types of technological change shape the joint evolution of structural change and time allocation in the model. The first elasticity is the one between c_g and c_s. In contrast to typical structural change models, these arguments of the utility function represent similar household needs that can either be satisfied by combining goods and time (burger at home) or services and time (burger at Mcdonald's). This interpretation suggests that this elasticity is larger than one. The other two key elasticities are those between time and either goods or services in the two home production functions. The difference between the two elasticities is key to determining the consumption shares of the two goods as income grows. As the opportunity cost of time increases with wage, the home production with the larger elasticity allows for easier substitutability of time with the other input. The differential technological change between the two home production technologies also plays a key role in generating structural change. If over time services allow to save more time in generating final consumption units, then the household might be willing to purchase a growing amount of them relative to goods even if their relative price is increasing. Finally, we note that in contrast with standard models (e.g. Ngai and Pissarides, 2007), the differential technological change between market production of goods and services reduces structural transformation. This is because, as goods become cheaper than services over time, and final consumption types are substitutes, the household has the incentive to increase the output of the home production that employs goods and reduce the output of the home production using services.

We calibrate the model to data on time allocation, value-added and prices of goods and services for the U.S. between 1965 and 2010. The model can account for the rise in the relative share of consumption of services via both increasing relative prices and quantities. It also generates a fall in market hours, hours spent in home production with goods, and hours spent in home production with services. This result is due to the calibrated value of key elasticities and the pattern of technological change. In line with the theory, the calibrated elasticity of substitution between c_g and c_s is larger than one. In addition, the elasticity of substitution between time and goods in home production is smaller than one while that between time and services is larger than one, which suggests that the household can more easily use services to save time but cannot as easily do the same by using goods. Finally, the calibration provides a measure of the differential rate of time-saving technological change between the two home production functions. This is faster in the home technology employing services. The difference implied by the calibration is around 1.43% per year. Thus, as wages increase due to growth in market productivity, the time of the household becomes more valuable. At the same time, since satisfying needs using services becomes easier in terms of time than doing so with goods, the household substitutes away final consumption obtained with goods for final consumption obtained with services, even though in the market the price of services is rising relative to the price of goods.

Finally, the model displays implications that are in line with cross-sectional evidence on consumption shares and wages. In U.S. data, richer consumers systematically display a larger share of services in consumption with respect to poorer ones, suggesting that a non-homothetic component in consumer preferences is needed to account for the data. Despite our theory not displaying non-homothetic preferences, the calibrated model shows a wage elasticity of services larger than one. We prove that this is due to the larger elasticity of substitution between services and time in home production with respect to the other home production function employing goods. As the wage of the household increases, so does the opportunity cost of time. As the home production with services allows for a larger substitutability between time and the other input, higher-wage households increase their purchase of services with respect to lower-wage ones. Thus, the calibrated model generates endogenous non-homotheticity.

Our modeling strategy is related to several insights described in the seminal work of Becker (1965) on the allocation of household time and its interaction with consumption. In this light, our setting is similar to those in Fang, Hannusch, and Silos (2021) and Bednar and Pretnar (2022). Both works, as us, assume that households derive utility from home activities that use time and goods and services purchased in the market. Fang, Hannusch, and Silos (2021) combine micro data on consumption expenditure and time use to estimate a model with only two activities: home production and leisure, in turn split between home luxuries and home necessities and leisure luxuries and leisure necessities, respectively. However, they do not study the implications of the model for structural change. Bednar and Pretnar (2022) study structural change in a context similar to ours. However, the interpretation of the model is different. While they see goods and services as associated with mutually exclusive activities, we see goods and services as being associated with highly substitutable activities. This assumption drives different interpretations of the model and induces an opposite estimated pattern of differential technological change between the two home production functions. Thus, only in our setting home-production with services allows the household to save time as the economy grows, in line with the intuition provided above.

2 Model

This section presents a simple model of structural transformation and time allocation among market work, home production, and leisure. While the model is static, we will allow technological levels to change over time in the quantitative part.

On the demand side, there is a representative household owning one unit of time and enjoying two types of final consumption: c_g and c_s. The consumption index of the household at time t is then given by

$$c_t = \left[(1-a)\, c_{g,t}^{\rho} + a c_{s,t}^{\rho}\right]^{\frac{1}{\rho}}.$$

Quantities of c_g and c_s cannot be directly obtained in the market. Instead, the household can produce them through two home-production technologies:

$$c_{g,t} = \left[(1-\alpha_g)\,(B_{g,t}h_{g,t})^{\rho_g} + \alpha_g q_{g,t}^{\rho_g}\right]^{\frac{1}{\rho_g}},$$

$$c_{s,t} = \left[(1-\alpha_s)\,(B_{s,t}h_{s,t})^{\rho_s} + \alpha_s q_{s,t}^{\rho_s}\right]^{\frac{1}{\rho_s}},$$

where q_g and q_s are quantities of goods and services that can be purchased in the market at prices p_g and p_s and h_g and h_s represent fractions of time.[1] We allow for levels of technology given by B_g and B_s, interpreting these as the productivity of home-production time. In the quantitative part, we allow these to grow over time to capture how, as the economy grows, the same amount of goods and services allows to produce the same amount of final consumption types c_g and c_s by using a reduced amount of time.

In our theory, the two types of consumption satisfy similar needs that can be fulfilled either with goods and time or with services and time. Given this interpretation of c_g and c_s, the elasticity of substitution governed by ρ is larger than one.[2] Also, we consider time as a more substitutable input in the home production using services than in the home production using goods, that is $\rho_g < \rho_s$.[3]

The objective of the household is to maximize her utility defined over final consumption c and leisure l:

$$u_t = \frac{c_t^{1-\sigma}}{1-\sigma} + \phi \frac{l_t^{1-\sigma}}{1-\sigma}. \tag{1}$$

[1]The objects c_g and c_s are called "activities" in Fang, Hannusch, and Silos (2021) and "consumption experience" in Bednar and Pretnar (2022). Here we simply refer to them as types of final consumption, to differentiate them from the intermediate use of q_g and q_s.

[2]Note, however, that in calibrating the model in section 3 we do not impose any restriction on ρ.

[3]Referring again to the burger example used above, we discussed how, over time, additional services allow to reduce the amount of time needed to consume a Big Mac. However, these additional services can be easily substituted with own time if the consumer chooses to purchase the Big Mac as in the 1960s by reaching the closest Mcdonald's by car.

The household sells the remaining time n (not used for either leisure or home production) in the market in exchange for a wage ω. Her budget constraint is then

$$p_{g,t}q_{g,t} + p_{s,t}q_{s,t} = \omega_t n_t,$$

while the time constraint is given by

$$n_t + h_{g,t} + h_{s,t} + l_t = 1.$$

On the supply side, there are two sectors in the market, one producing goods and one producing services. There is a price-taking firm acting in perfect competition in each sector that uses labor time to produce output using the following technology:

$$Y_{i,t} = A_{i,t}L_{i,t},\ i = g, s, \tag{2}$$

where A_i is the technological level of the firm and L_i is the amount of time used in production. The firm in each sector solves the following profit-maximization problem

$$\max_{L_{i,t}} \pi_{i,t} = p_{i,t}\left(A_{i,t}L_{i,t}\right) - \omega_t L_{i,t}. \tag{3}$$

3 Calibration

3.1 Time Use Data

We use the 1965-1966 Americans' Use of Time and the 2010 American Time Use Survey to estimate the amount of time households spent on different activities.[4] Table (1) summarizes our results. The most extensive definition of market work includes core working time, travel, activities related to work (e.g. eating at work, coffee breaks, socializing) and activities related to job search. This measure displays an average reduction of six hours per week between 1965 and 2010. The bulk of it is core work, which fell by almost three hours on average. Including travel time, market work is further reduced by half an hour per week. Since other work-related activities include items that can arguably be defined as leisure at work, we use core work plus commuting time as our benchmark measure of market work time.

We depart from previous work by dividing home-production time into two separate components. The first one includes activities in which the individual uses goods as inputs and could reasonably be substituted with a market-purchased service.[5] This includes meal prepa-

[4] See Appendix A.1 for details.

[5] This can be thought as a typical home-production function in capital and labor appearing in several

Table 1: Hours per week spent in market work and home production

Time use category	**Average hours per week**		
	1965	**2010**	**Difference**
Market work	35.88	29.82	-6.06
Core work and travel	32.42	29.13	-3.30
Home production, goods	20.09	17.16	-2.93
Incl. childcare	23.76	22.61	-1.16
Incl. childcare and other care	24.35	23.92	-0.44
Home production, services	1.97	0.52	-1.44
Incl. education	3.38	1.85	-1.53

ration and clean-up, home and car maintenance and other household chores. Childcare and other types of care for others (mainly adult care) could also be placed in this category. These arguably have a leisure component but can also be substituted with market services. Our baseline measure of time employed in home-production with goods excludes care time, as it is not necessarily associated with the utilization of goods. This measure experienced a reduction of around three hours per week during our period of interest. If we include time used caring for others, the reduction is of only half an hour per week on average, and the difference is mainly due to increased childcare hours.

The other home-production category includes activities in which market services are used as input.[6] This is the smallest category quantitatively but experienced the largest proportional reduction. Our baseline measure includes time spent procuring services, which declined from two hours to half an hour per week between 1965 and 2010. Education time could arguably be placed here too. However, we consider this type of time use as a form of investment which does not provide immediate consumption for the household and so we do not include it in our definition of home production time with services.

Considered together, our preferred measures of time spent in market work, home production with goods and home production with services implies an increase of 7.67 weekly leisure hours on average.

structural change models, with a broad definition of goods replacing the notion of capital. See for instance Ngai and Pissarides (2008) and Moro, Moslehi, and Tanaka (2017) among many.

[6]Thus, our framework can also be interpreted as one in which search frictions to acquire services in the market are reduced over time.

3.2 Strategy

Since the production functions of the firms are linear in labor time, the price of services relative to goods in the model is given by the inverse of the relative productivities. We normalize this to 1 in 1965 and retrieve the growth rates by constructing value-added per hour worked in each of the two sectors. We obtain average yearly productivity growth rates of 0.79% and 1.31% in the services and goods sectors, respectively.[7]

We calibrate the rest of the parameters in the model simultaneously by minimizing the sum of the squares of the per cent distance between a set of data targets and the corresponding model's counterparts. There are two subsets of targets for a total of nine data targets. The first subset of targets includes statistics for the U.S. economy from the initial year of the sample and pins down the parameters in the set $\{a, \phi, \alpha_s, \alpha_g\}$: the share of services in expenditure, and the average time allocation (hours per week spent in market work, home production with goods and home production with services). The second subset of targets includes the changes in all of the aforementioned statistics between the initial and final years, plus the change in the real quantity of services and pins down the parameters that govern the different elasticities present in the model: $\{\sigma, \rho, \rho_s, \rho_g\}$. Finally, we impose that there is no productivity growth in the home production employing goods, $\gamma_{B_g} = 0$. Thus, productivity growth in the home production with services, γ_{B_s}, also provides a measure of the differential productivity growth between the two home sectors, which is the main measure we are interested in. A detailed description of the calibration is reported in Appendix B.

4 Quantitative Analysis

4.1 Results

Table (2) reports the fit of the calibration and the implied parameter values. In general, the fit of the model is good, both in terms of structural transformation and time allocation. The calibrated model generates an increase in the relative share of consumption of services that is comparable to the data (0.22 in the model versus 0.20 in the data). Importantly, as in the data, this increase is driven by both an increase in the relative price and in the relative quantity of services. The relative price of services to goods in the model is entirely driven by relative TFP in the two market sectors, which increases by 17.8% in both model and data. The growth of the quantity of goods is not large over the period considered: it displays a growth factor of -6.6% in the model and of 6.5% in the data.[8] The quantity of

[7] See Appendix A.2 for details on the data used for market sectors.

[8] This is not reported in Table (2) because it is not used as a target in the calibration.

Table 2: Calibrated parameters and targets

Parameter	**Value**	**Target**	**Model**	**Data**
Economy in 1965				
ϕ	17.339	Market work hours/week	29.735	32.422
α_s	0.683	Home production hours/week with services	1.807	1.968
α_g	0.386	Home production hours/week with goods	18.165	20.090
a	0.378	Services share in expenditure	0.465	0.482
Changes between 1965 and 2010				
σ	3.301	Δ market work hours/week	-3.593	-3.295
ρ_g	-2.488	Δ home production hours/week with services	-1.365	-1.444
ρ_s	0.623	Δ home production hours/week with goods	-3.686	-2.933
ρ	0.443	Δ services share in expenditure	0.217	0.195
γ_{B_s}	0.014	Δ real quantity of services	1.843	2.147

services instead grows substantially, displaying a 1.84 factor increase in the model and a 2.15 in the data. Thus, the calibrated model accounts for 86% of the growth in the quantity of services over time. The top panels of Figure (1) provide a visual fit of the model over the period, assuming constant yearly growth rates of technology reported in Table (2).

In terms of time allocation, the model generates a similar fall in hours worked and the corresponding increase in leisure observed in the data: a decline of 1.37 hours of home work with services compared to 1.44 in the data, of 3.69 hours of home-work with goods compared to 2.93 in the data, and of 3.59 hours of market work compared to a decline of 3.30 hours in the data.

The key parameter values display an estimated sign that is in line with the theory. First, the elasticity of substitution between final consumption types c_s and c_g is larger than one (1.8), consistent with the idea that they satisfy similar needs. This result is similar in sign and magnitude to the estimated elasticity in Fang, Hannusch, and Silos (2021) (2.628) obtained using a pseudo-panel that matches time use and consumption expenditure data for different education groups of the population.[9]

Second, we find that time and goods are imperfect complements in the home production of c_g (with an elasticity of 0.29), while they are imperfect substitutes in the home production of c_s (elasticity of 2.65). This result supports the view that in obtaining final consumption, it is easier for the household to substitute her own time with services in the market rather than substituting it with goods.

[9]It must be noted, however, that they consider a model in which goods and services are used also in the production of leisure, so their estimated value is over two types of final consumption and two types of final leisure.

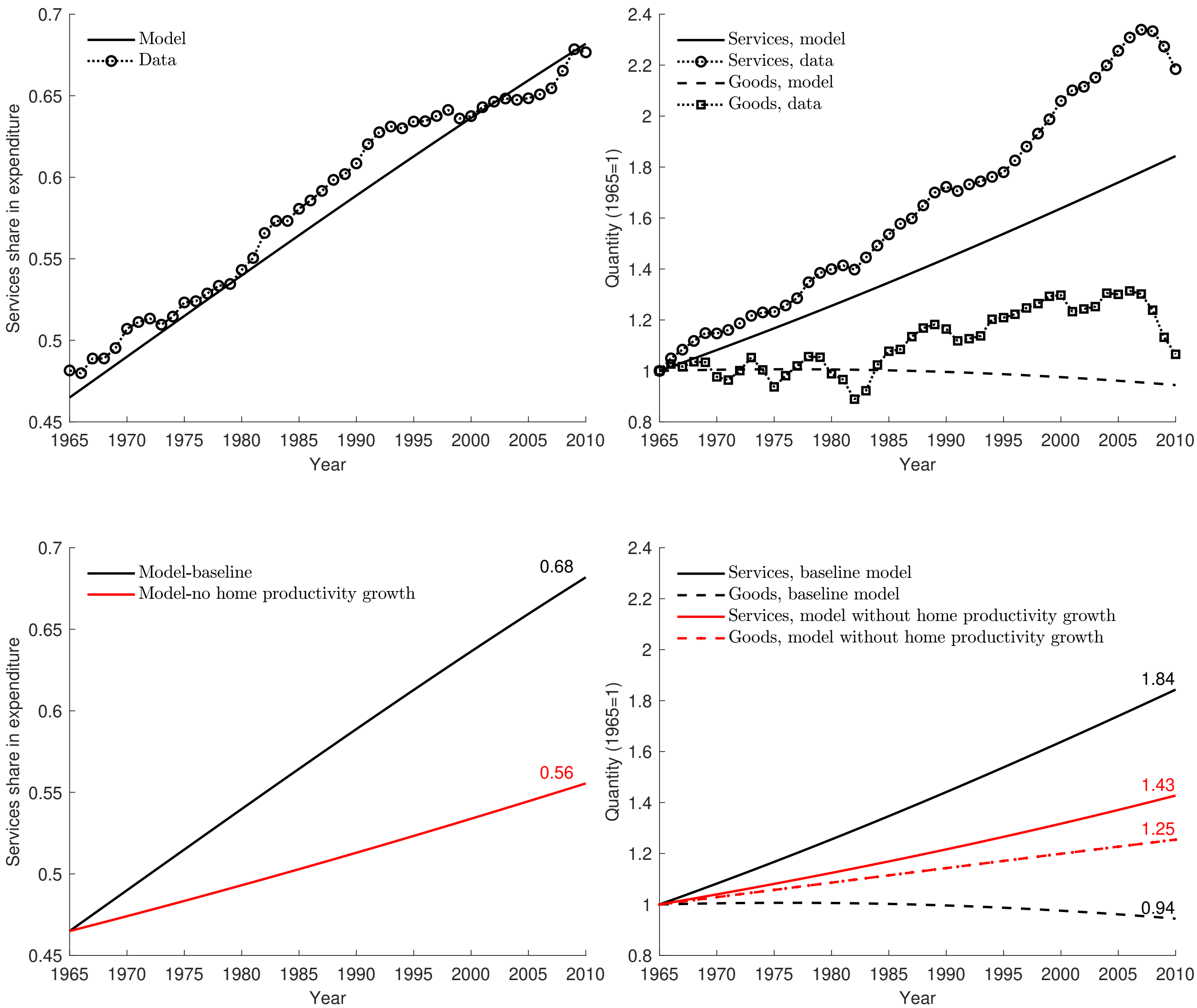

Figure 1: Structural change in the data and in the model: relative expenditures and quantities of services and goods.

Third, the calibration provides a measure of the differential rate of technological change between the two home production functions of 1.4% per year. Thus, technological change is faster in the home technology employing services than in that employing goods as inputs. This difference captures the fact that over time employing time with services allows one to obtain final consumption more efficiently than employing time with goods.[10]

[10]This result is opposite to Bednar and Pretnar (2022), who find an increase in the efficiency of home production with goods relative to home production with services. There are two main reasons behind this difference. First, they assume that technological change in the home production function is Hicks-neutral, thus augmenting simultaneously the productivity of time and the other input. Here we assume that technological change magnifies the productivity of time, affecting the other input only through marginal productivity. Second, following the intuition provided in the example in the introduction, we interpret the

The estimated parameters concur in shaping structural change in the model through two mechanisms. The first one works through time-saving technological change. As time passes, the price of services increases faster than that of goods. However, there is a concurrent increase in efficiency in the home production function using services relative to goods, which allows a larger amount of time-saving. As wages increase due to growth in market productivity, the opportunity cost of time increases, and such time saving compensates for the increase in the relative price of services, making the home production with services more efficient (i.e. less costly) with respect to the home production with goods. As a result, the household substitutes away goods consumption for services consumption, even though in the market, the price of services is rising relative to the price of goods.

The second mechanism works through the emergence of endogenous non-homothetic effects in equilibrium, which are due to the difference in the estimated elasticities between time and the other input across the home production function. In the next section, we discuss in detail these two mechanisms.

4.2 The contribution of time-saving technological change

One channel generating structural transformation in our model is the differential pace of technological change in the two home production functions. In particular, the calibration delivers an increase in B_s/B_g over time, suggesting that the type of home-production that uses services allows for a larger time saving as the economy grows.

We then ask how much of the structural change generated in the model is due to the estimated increase in B_s/B_g. To do this, we run a counterfactual in which we set productivity growth in the home services sector to zero. In this way, we remove the technological factor that makes over time production at home with services more efficient with respect to producing at home with goods.

The bottom panels of Figure (1) show the model's outcomes when the growth of both B_s and B_g is set to zero and compare these with the benchmark case.[11] The increase in expenditure on services relative to goods is roughly half that in the baseline calibration (from 0.47 to 0.56 instead of 0.47 to 0.68). The bottom-right panel of Figure (1) shows that the smaller extent of structural change is driven by lower growth in the real quantity of services and higher growth in the real quantity of goods over time. Thus, without the

two types of final consumption as satisfying similar needs, which implies an elasticity of substitution larger than one, while they assume that c_g and c_s are associated with mutually exclusive activities, which implies an elasticity of substitution smaller than one. In the calibration, we do not impose any restriction in the estimation of ρ and find that the two activities are substitutes. See Appendix B for details.

[11] Recall that in the benchmark calibration the growth of B_g is restricted to be zero, and only B_s displays positive growth.

efficiency gain due to the growth of B_s/B_g, the household is less willing to use the home production function employing services and structural change is substantially reduced.

4.3 Income effects

In the previous sections, we show that i) our theory can account for structural transformation in the aggregate time series when allowing for a faster pace of time-saving technology in home production with services and that ii) in a counterfactual that sets technological change in both types of home productions to zero there is still a substantial amount of structural transformation. We note here that the remaining part of structural transformation cannot be due to relative productivity changes in the market, i.e. the growth in A_g/A_s. Indeed, the growth of A_g/A_s dampens structural change in our model by making goods cheaper than services. This implies that, *ceteris paribus*, the type of home production using goods becomes less costly with respect to that using services. As a result, because final consumption types are substitutes, this induces an increase in the utilization of goods relative to services. It follows that the residual structural change observed in the bottom panels of Figure (1) must be generated through some other channel, not related to technological change. This section shows that this channel emerges in the model through endogenous income effects.

To do this, we study how the implicit cost of home production changes as the wage of the household changes (i.e. in a cross-section of identical agents except for their wage). As time is an input in home production functions, a larger wage implies a higher cost of that input in production activities. Thus, households at higher wage levels face a larger implicit cost of home production activities with respect to households at lower wage levels. However, the increase in the cost of the two home activities might not be the same, as it depends on the specific elasticity of substitution between time and the other input, resulting in an observed non-homothetic behavior of market-purchased inputs.

To show the above point, we consider a generic home-production activity as represented by a CES production function in time and another input (services or goods). Home-production can then be represented as

$$y = \bar{y}\left[(1-\bar{\alpha})\left(\frac{h}{\bar{h}}\right)^{\bar{\rho}} + \bar{\alpha}\left(\frac{q}{\bar{q}}\right)^{\bar{\rho}}\right]^{\frac{1}{\bar{\rho}}}, \tag{4}$$

where $1/(1-\bar{\rho})$ is the elasticity of substitution between time h and the other input q. The CES function in (4) is expressed in "normalized" form. This is because we are interested in investigating the behavior of the cost function associated with (4) when the elasticity of substitution between inputs changes (i.e. when $\bar{\rho}$ changes). Without normalization, it is impossible to make meaningful comparisons, as highlighted in La Grandville (2009), León-Ledesma, McAdam, and Willman (2010), Cantore, León-Ledesma, McAdam, and Willman (2014), among others. Here $\bar{h}$, $\bar{q}$ and $\bar{y}$ are the values of h, q and production y at the normalization point and $\bar{\alpha}$ is the share of output that goes to input q at the normalization point. This is given by

$$\bar{\alpha} = \frac{\bar{p}\bar{q}}{\bar{p}\bar{q} + \bar{w}\bar{h}}$$

where $\bar{w}$ and $\bar{p}$ are the prices of the two inputs at the normalization point.

Using a change of variable we can then write $\widetilde{y} = \frac{y}{\bar{y}}$ $\widetilde{h} = \frac{h}{\bar{h}}$ and $\widetilde{q} = \frac{q}{\bar{q}}$ so that the normalized CES becomes

$$\widetilde{y} = \left[(1-\bar{\alpha})\left(\widetilde{h}\right)^{\bar{\rho}} + \bar{\alpha}\left(\widetilde{q}\right)^{\bar{\rho}}\right]^{\frac{1}{\bar{\rho}}}. \tag{5}$$

We now assume that a firm acting in perfect competition wants to maximize (5) subject to the cost constraint

$$\widetilde{w}\widetilde{h} + \widetilde{p}\widetilde{q} \leq \widehat{C}.$$

Here $\widetilde{p}$ and $\widetilde{w}$ are the market prices expressed in some currency of units of $\widetilde{h}$ and $\widetilde{q}$, and $\widehat{C}$ is the total cost in that currency of purchasing quantities $\widetilde{h}$ and $\widetilde{q}$. From the first order conditions of the profit maximizing firm, we obtain

$$\frac{\widetilde{p}\widetilde{q}}{\widetilde{w}\widetilde{h}} = \frac{\bar{\alpha}}{1-\bar{\alpha}}\left(\frac{\widetilde{q}}{\widetilde{h}}\right)^{\bar{\rho}}.$$

At the normalization point, we have that both $\widetilde{h}$ and $\widetilde{q}$ are equal to one, so

$$\frac{\widetilde{p}\widetilde{q}}{\widetilde{w}\widetilde{h}} = \frac{\bar{\alpha}}{1-\bar{\alpha}}$$

and also

$$\frac{\widetilde{p}}{\widetilde{w}} = \frac{\bar{\alpha}}{1-\bar{\alpha}}.$$

Thus, at the normalization point, the value of the parameter $\bar{\alpha}$ determines the shares of the two factors in output and the relative price of the two inputs.

The cost function corresponding to the production function in (4) is

$$\widetilde{C} = \left[(1-\bar{\alpha})^{\frac{1}{1-\bar{\rho}}}\left(\widetilde{w}\right)^{\frac{\bar{\rho}}{\bar{\rho}-1}} + \bar{\alpha}^{\frac{1}{1-\bar{\rho}}}\left(\widetilde{p}\right)^{\frac{\bar{\rho}}{\bar{\rho}-1}}\right]^{\frac{\bar{\rho}-1}{\bar{\rho}}}.$$

Consider the following point of normalization $\bar{h} = 1/2$, $\bar{q} = 1/2$ and assume that the output share of factor q is $1/2$ at the normalization point

$$\bar{\alpha} = \frac{\widetilde{p}\widetilde{q}}{\widetilde{w}\widetilde{h} + \widetilde{p}\widetilde{q}} = 1/2.$$

From this, it follows that it must be that $\widetilde{w}/\widetilde{p} = 1$ at the normalization point. Finally, we want to express the cost function in currency units that make $\widetilde{C} = 1$ at the normalization point.[12] To have this, let's start with a guess for the two prices that satisfies the restriction $\widetilde{w}/\widetilde{p} = 1$. Let's impose $w = p = 1$. Then we have

$$C = w\widetilde{h} + p\widetilde{q} = 1 * 1 + 1 * 1 = 2.$$

The cost function is not equal to 1 at the normalization point. We can then change the currency in which prices and costs are given and divide all prices by 2.

$$\widetilde{C} = \widetilde{w}\widetilde{h} + \widetilde{p}\widetilde{q} = 1/2 * 1 + 1/2 * 1 = 1.$$

Thus, at our normalization point, we have $\widetilde{q} = \widetilde{h} = 1$, $\widetilde{w} = \widetilde{p} = 1/2$ and $\widetilde{C} = 1$:

$$\widetilde{C} = \left[(1-\bar{\alpha})^{\frac{1}{1-\bar{\rho}}} (1/2)^{\frac{\bar{\rho}}{\bar{\rho}-1}} + \bar{\alpha}^{\frac{1}{1-\bar{\rho}}} (1/2)^{\frac{\bar{\rho}}{\bar{\rho}-1}}\right]^{\frac{\bar{\rho}-1}{\bar{\rho}}} = 1.$$

This expression holds for any value of $\bar{\rho}$.[13] This means that the derivative of these expressions with respect to $\bar{\rho}$ is always zero. Taking the derivative of the cost function with respect to

[12] We can always do this as we can divide all prices to a suitable numeraire to obtain the result.

[13] Our analysis does not depend on a specific normalization point chosen. Assume, for instance, that the output share of factor q is $1/3$ at the normalization point

$$\bar{\alpha} = \frac{\widetilde{p}\widetilde{q}}{\widetilde{w}\widetilde{h} + \widetilde{p}\widetilde{q}} = 1/3.$$

From this it follows that it must be that $\widetilde{w} = 2\widetilde{p}$. We want to express the cost function in currency units that make $\widetilde{C} = 1$ at the normalization point. Let's guess that that $\widetilde{w}$ is the numeraire of the economy. Then, using the restriction $\widetilde{w}/\widetilde{p} = 2$, we have

$$C = w\widetilde{h} + p\widetilde{q} = 1 * 1 + 1/2 * 1 = 3/2.$$

Let's divide prices by $3/2$ to obtain

$$\widetilde{C} = \widetilde{w}\widetilde{h} + \widetilde{p}\widetilde{q} = 1 * 2/3 * 1 + 1/2 * 2/3 * 1 = 1.$$

So the two prices at the normalization point are $\widetilde{w} = 2/3$ and $\widetilde{p} = 1/3$. Thus, at our normalization point we have $\widetilde{q} = \widetilde{h} = 1$, $\widetilde{w} = 2/3$, $\widetilde{p} = 1/3$ and $\widetilde{C} = 1$. All the results presented in this section hold for this alternative normalization point.

$\widetilde{w}$ we obtain

$$\widetilde{C}'_{\widetilde{w}} = \widetilde{C}^{-\frac{1}{1-\bar{\rho}}} (1-\bar{\alpha})^{\frac{1}{1-\bar{\rho}}} \widetilde{w}^{\frac{1}{\bar{\rho}-1}}.$$

Evaluating this at the normalization point and noting that $\widetilde{C} = 1$ at that point, we are left with

$$\widetilde{C}'_{\widetilde{w}} = (1-\bar{\alpha})^{\frac{1}{1-\bar{\rho}}} \widetilde{w}^{\frac{1}{\bar{\rho}-1}}.$$

Taking the derivative of this with respect to $\bar{\rho}$

$$\widetilde{C}'_{\widetilde{w}\bar{\rho}} = (1-\bar{\alpha})^{\frac{1}{1-\bar{\rho}}} \widetilde{w}^{\frac{1}{\bar{\rho}-1}} log\,(1-\bar{\alpha}) \left(\frac{1}{(1-\bar{\rho})^2}\right) +$$

$$+ (1-\bar{\alpha})^{\frac{1}{1-\bar{\rho}}} \widetilde{w}^{\frac{1}{\rho\bar{\rho}-1}} log\,(\widetilde{w}) \left(\frac{-1}{(\bar{\rho}-1)^2}\right) = 0$$

The last expression shows that at the normalization point, there is no effect of a change in $\bar{\rho}$ on the variation in the total cost induced by a change in w. However, this result holds only for an infinitesimal increase of w, while for discrete increments, the behavior of the cost function depends on the value of $\bar{\rho}$. This is reported in Figure (2). As the wage increases, the cost increases faster the smaller the elasticity of substitution, which is governed by $\bar{\rho}$. The rationale for this result is at the heart of our theory. When the wage increases, the opportunity cost of time also rises, and the household is induced to optimally reduce time and use more of the other input due to a substitution effect. However, the extent to which this is possible depends on the elasticity of substitution between the two inputs: when the elasticity is larger, it is easier to substitute time with the other input. As a result, the increase in total cost induced by an increasing wage is smaller when the elasticity of substitution is larger.

Our calibration delivers an elasticity of substitution between time and services of 2.65 and between time and goods of 0.29. Thus, according to the analysis in this section, the model predicts a larger ratio of the quantity of services over that of goods for households with higher wages. This amounts to a wage elasticity of services larger than the wage elasticity of goods, resulting in a rising share of services in total expenditure on goods and services. In Appendix C we provide an assessment of the quantitative relevance of income effects in the calibrated model.

5 Conclusion

We present a new theory of structural transformation that can successfully reproduce structural change and time allocation patterns in the U.S. time series, and which also provides a

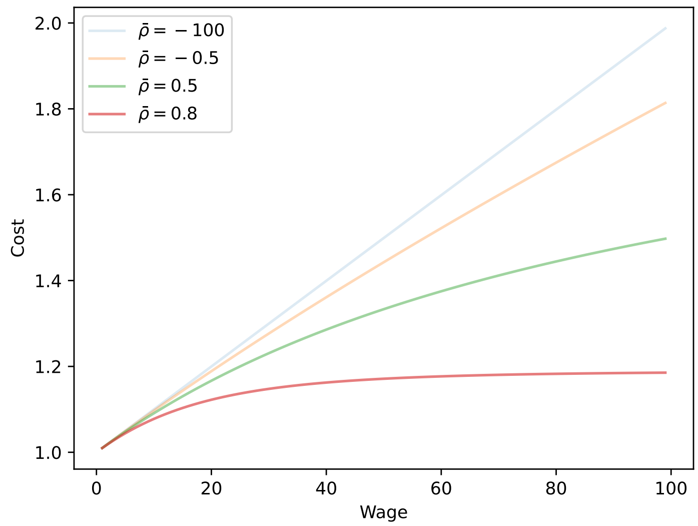

Figure 2: Total cost of home-production as wage increases for different values of $\bar{\rho}$.

qualitative prediction in line with cross-section data, i.e. a share of services in consumption that grows with the wage of the agent. Our setting is simple and displays only homothetic functions. This makes it suitable to be employed in settings where a non-homothetic behavior is needed without resorting to non-homothetic utility functions, which are generally less tractable. In addition, our theory could be tested using cross-country data on time use to investigate to what extent, technological change that makes home production with services more efficient and endogenous income effects are responsible for structural change observed over the development path.

A Data

A.1 Time Use

We follow Aguiar and Hurst (2007) in selecting the sample (people aged 21-65, excluding students and retirees), constructing weights that keep the demographic composition constant and categorizing activities consistently across surveys. Starting from this categorization, we assume 70 hours per week are needed for essential sleeping, eating and personal care. We assign the remaining 98 hours to market work, home production with goods, home production with services and leisure. Table 3 shows the classification of activities into five time use components: essential, leisure, market work, home production with goods and home production with services. Activities in the leftmost column roughly coincide with those defined by Aguiar and Hurst (2007).

A.2 Market Sectors

Data on hours worked in the service sector is taken from the CPS March supplement. We use the same methodology as Ngai and Petrongolo (2017) to classify industries and obtain the total hours worked for each person and year before aggregating.

Finally, we use the ten-sector database of the Groningen Growth and Development Centre to obtain nominal and constant dollars value added for the U.S. Consistent with the definition in Ngai and Petrongolo (2017) we aggregate 1) Agriculture, 2) Mining, 3) Manufacturing, 4) Utilities, 5) Construction into the goods sector, and 6) Trade, restaurants and hotels, 7) Transport, storage and communication, 8) Finance, insurance, real estate and business services 9) Government services 10) Community, social and personal services into the services sector. These series are the data counterparts of goods and services purchased in the market in the model.

To compute the growth rate of market productivity in the production of services and goods, we proceed as follows: first, we obtain the time series for the total number of hours worked in the United States (hours worked by full-time and part-time employees from the BEA). We then use the methodology in Ngai and Petrongolo (2017) and data from the CPS to obtain estimates of the fraction of hours worked in the services and the goods sectors. Using both data series, we estimate the total number of hours worked per year in each of the two sectors and divide the real value added by that number. We obtain that the annualized growth rate in the value added per hour worked (our measure of the growth rate in productivity) is 1.68% in the goods sector and 0.8% in the services sector.

Table 3: Classification of activities

Activity	**Baseline time allocation**
Child care	
Child care basic	Leisure
Child care teach	Leisure
Child care play	Leisure
Eating, sleeping and personal care	
Eating	Essential
Sleeping	Essential
Personal care	Essential
Own medical care	Leisure
Other care	Leisure
Home work, shopping, etc.	
Meals	Home production/goods
Housework	Home production/goods
Home care maintenance	Home production/goods
Home other	Home production/goods
Garden pet	Home production/goods
Shopping for goods	Home production/goods
Shopping for services	Home production/services
Market work	
Work travel	Market work
Work related	Leisure
Work core	Market work
Work unemp.	Market work
Education	Leisure
Civic	Leisure
Leisure	
Exercise sports	Leisure
TV	Leisure
Socializing	Leisure
Reading	Leisure
Ent. Not TV	Leisure
Hobbies	Leisure

B Calibration

Except for the growth of TFP in market sectors, we calibrate the parameters in the model simultaneously by minimizing the sum of the squares of the per cent distance between a set of data targets and the corresponding model's counterparts. There are two subsets of targets for a total of nine data targets.

The first subset of targets includes statistics for the U.S. economy from the initial year of the sample and pins down the parameters in the set $\{a, \phi, \alpha_s, \alpha_g\}$: the share of services in expenditure, and the average time allocation (hours per week spent in market work, home production with goods and home production with services). Since a is the weight of the consumption produced with services in overall consumption, a higher (lower) value for a implies a higher (lower) share of services in expenditure in the initial year. Moreover, being the weight of leisure in utility, the value of ϕ affects directly the amount of time the household spends working. Finally, α_s and α_g are the weights of services and goods in each home production technology, and thus affect negatively the amount of time spent in home production with services and goods, respectively.

The second subset of targets includes the changes in all of the aforementioned statistics between the initial and final years, plus the change in the real quantity of services and pins down the parameters that govern the different elasticities present in the model: $\{\sigma, \rho, \rho_s, \rho_g\}$. The elasticity between leisure and consumption depends on σ. Substitutability increases as σ approaches zero, which causes leisure to be more responsive to changes in wages. The elasticity of substitution between consumption produced with services and consumption produced with goods is determined by $\rho \in (-\infty, 1]$ while ρ_s and ρ_g determine the elasticity of substitution between time and services or goods as inputs in the home production of the two types of consumption. Together, ρ, ρ_s, and ρ_g determine the changes in home production times and the expenditure share in services between the initial and final period. Finally, we impose that there is no productivity growth in the home production employing goods, $\gamma_{B_g} = 0$. Thus, productivity growth in the home production with services, γ_{B_s}, also provides a measure of the differential productivity growth between the two home sectors, which is the main measure we are interested in. As γ_{B_s} directly affects the growth in the real quantity of services purchased by the household, we use this moment to pin it down.

We run the minimization algorithm from 1000 different starting points and keep the results of convergence only when the value of the objective function is below a certain threshold (136 runs). Some of these runs include implausibly high values for ϕ, so we drop them. We end up with 103 sets of parameters. Figure (3) shows the kernel density estimation of the distribution and the average for each of the nine parameters calibrated by matching the

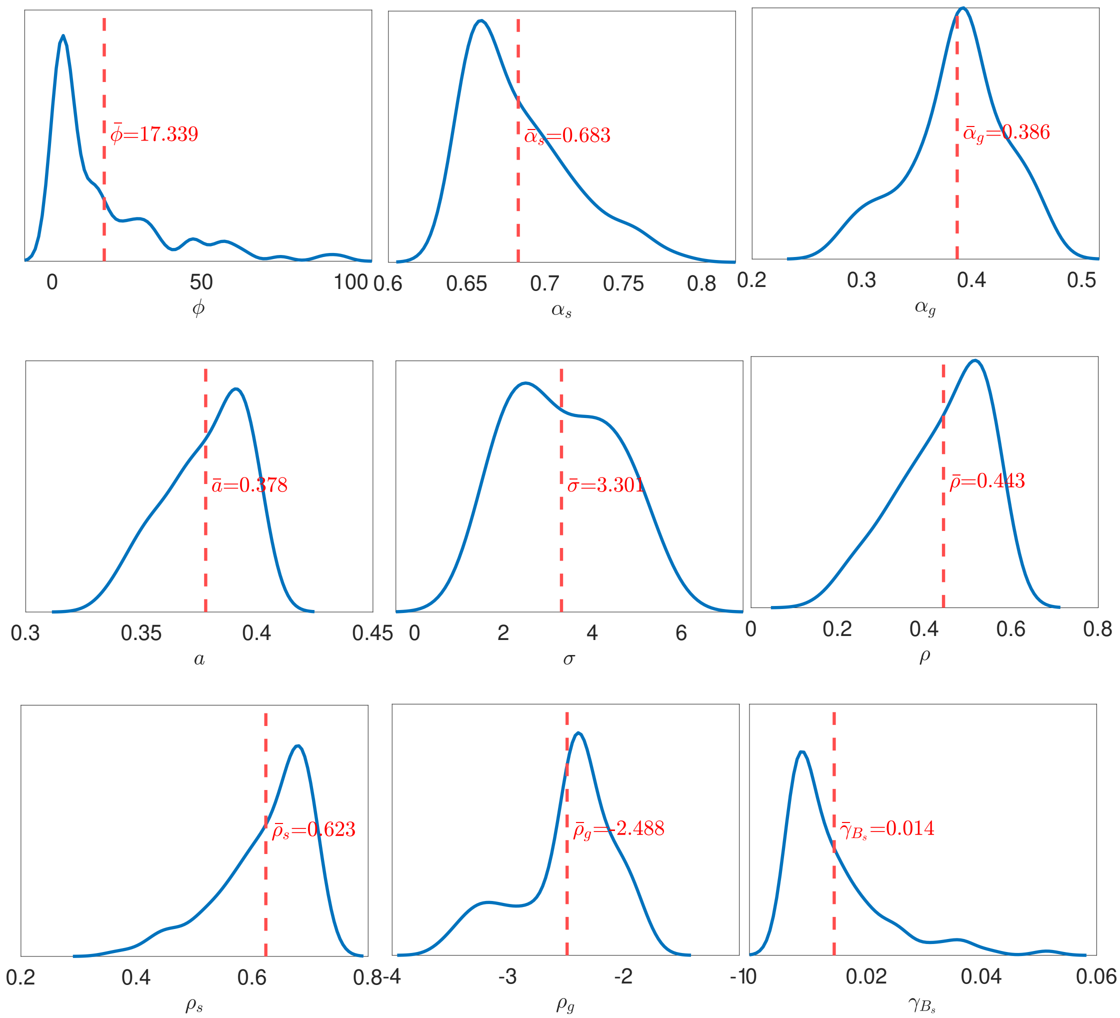

Figure 3: Distributions of calibrated parameters

moments from the data for the 103 sets of results. In the benchmark calibration in the text, we report the mean of each parameter.

C Services share in the cross-section

Typically, an increase in the services share with income or expenditure is observed in cross-sectional data (i.e. with fixed relative prices). The literature discusses this as being the reason a non-homothetic component is needed for structural change models to fit the data (Boppart, 2014, Buera, Kaboski, Rogerson, and Vizcaino, 2022 and Alder, Boppart, and Muller, 2022). In section 4.3 we studied why income effects emerge when the elasticity of substitution between time and the other inputs is different across home production technologies. These effects allow the model to generate structural transformation in Figure (1) even though B_s/B_g

does not change over time and the fact that the growth in A_g/A_s contributes to producing reversed structural change. This suggests that income effects generated by the model are substantial. In this Appendix we are then interested in assessing the performance of the model in generating income effects in a cross-section of individuals that differ only in their wage level.

We first follow the methodology in Boppart (2014) and use CEX interview data obtained from the ICPSR for the year 2003 to calculate the median income across households and the fractions of such income associated with each quintile of the distribution.[14] We then take the household problem described in section 2, and parametrize it using the calibrated values from Table (2) and the values of prices that emerge in the general equilibrium of the calibrated model in the year 2010. We then perform the following exercise. We assume that the general equilibrium wage corresponds to the median income in the data. We solve the household problem for a range of fractions of the general equilibrium wage that match the fractions of the median income given by each income quintile in the data. Figure (4) reports the share of services in consumption of households with different wages in the model and the corresponding income measure for the data. The model reproduces the positive relationship between wages/income and share of services, although the magnitude is substantially larger with respect to the data.

The endogenous income effect is generated by the difference in the elasticity of substitution between the two home production functions that are delivered by the calibration. If we assume identical home production functions, the Engel curve of services relative to goods does not display any slope.[15]

[14] As described in Boppart (2014) the "quintiles refer to total household labor earnings after taxes plus transfers per OECD modified equivalence scale. For homeowners, the imputed renting value is taken as shelter expenditures."

[15] We report results when both functions have the same parametrization as the goods home production function in Table 2. In addition, we run counterfactuals to investigate whether, in addition to differences in ρ_g and ρ_s, also differences in α_g and α_s play a role in generating income effects. We find that setting the same α in the two home production functions, while keeping all the other parameters from the benchmark calibration, only produces a level effect but does not affect the magnitude of the patterns of structural change.

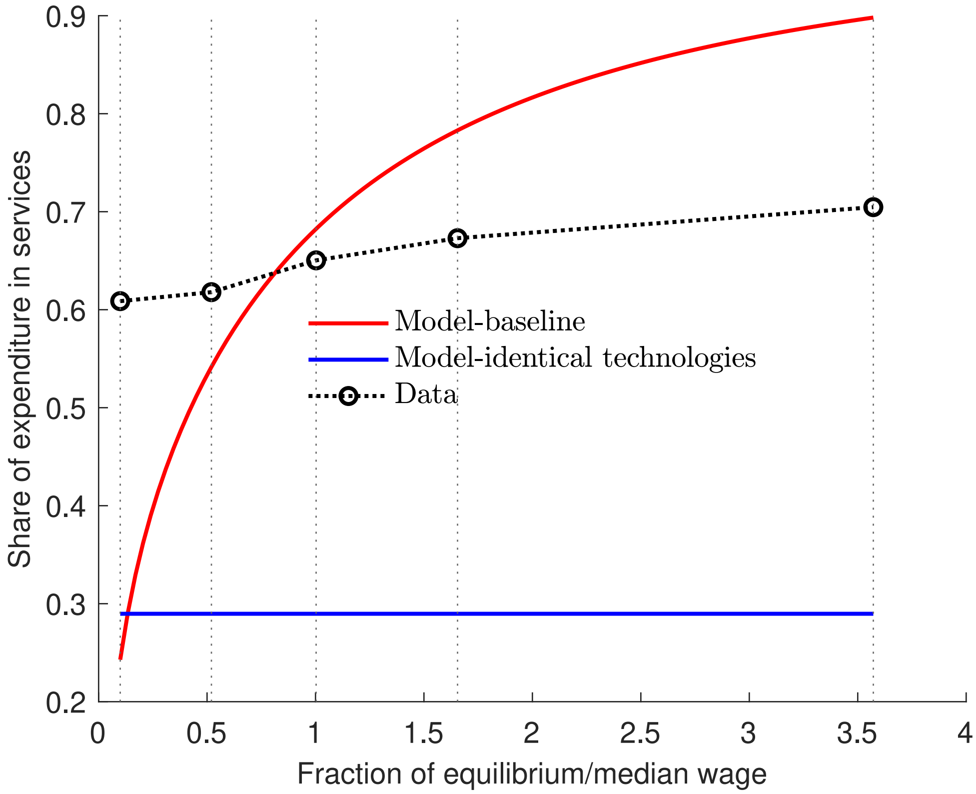

Figure 4: Share of expenditures in services with fixed relative prices and varying wages

References

Aguiar, M., and E. Hurst (2007): "Measuring trends in leisure: The allocation of time over five decades," *The quarterly journal of economics*, 122(3), 969–1006.

Alder, S., T. Boppart, and A. Muller (2022): "A Theory of Structural Change That Can Fit the Data," *American Economic Journal: Macroeconomics*, 14(2), 160–206.

Becker, G. S. (1965): "A Theory of the Allocation of Time," *The economic journal*, 75(299), 493–517.

Bednar, B., and N. Pretnar (2022): "Structural Change Under Home Production with Time to Consume," Discussion paper, working paper.

Boppart, T. (2014): "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences," *Econometrica*, 82(6), 2167–2196.

Bridgman, B. (2016): "Home productivity," *Journal of Economic Dynamics and Control*, 71, 60 – 76.

Buera, F. J., and J. P. Kaboski (2009): "Can Traditional Theories of Structural Change Fit the Data?," *Journal of the European Economic Association*, 7(2-3), 469–477.

Buera, F. J., J. P. Kaboski, R. Rogerson, and J. I. Vizcaino (2022): "Skill-biased structural change," *The Review of Economic Studies*, 89(2), 592–625.

Cantore, C., M. León-Ledesma, P. McAdam, and A. Willman (2014): "Shocking stuff: technology, hours, and factor substitution," *Journal of the European Economic Association*, 12(1), 108–128.

Comin, D., D. Lashkari, and M. Mestieri (2021): "Structural change with long-run income and price effects," *Econometrica*, 89(1), 311–374.

Fang, L., A. Hannusch, and P. Silos (2021): "Luxuries, necessities, and the allocation of time," Discussion paper, Working Paper.

Kongsamut, P., S. Rebelo, and D. Xie (2001): "Beyond Balanced Growth," *The Review of Economic Studies*, 68(4), 869–882.

La Grandville, O. (2009): "Economic growth: A unified approach," .

León-Ledesma, M. A., P. McAdam, and A. Willman (2010): "Identifying the Elasticity of Substitution with Biased Technical Change," *The American Economic Review*, 100(4), 1330–1357.

Moro, A., S. Moslehi, and S. Tanaka (2017): "Does Home Production Drive Structural Transformation?," *American Economic Journal: Macroeconomics*, 9(3), 116–46.

Ngai, L. R., and B. Petrongolo (2017): "Gender gaps and the rise of the service economy," *American Economic Journal: Macroeconomics*, 9(4), 1–44.

Ngai, L. R., and C. A. Pissarides (2007): "Structural Change in a Multisector Model of Growth," *The American Economic Review*, 97(1), 429–443.

——— (2008): "Trends in Hours and Economic Growth," *Review of Economic Dynamics*, 11(2), 239–256.