

Online Appendix to “The Rise of Services and Balanced Growth in Theory and Data”

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A Capital services and capital stock

This part of the Appendix addresses the issue of whether capital services or the capital stock should be used in the production function. For this, we follow closely Jorgenson and Griliches [1967], Jorgenson et al. [1987], Hall and Jorgenson [1967], and Oulton and Srinivasan [2003] amongst many others.

A.1 Concepts

We first make some definitions and settle notation that will be used in what follows:

- The wealth concept of aggregate capital, that is, capital assets available in the NIPA assets accounts, is the sum of the values of different asset stocks. These asset stocks are the perpetual inventory value of past investments. Productive capital stock is the sum of the value of those assets that can be used to produce output. We will refer to this as “**wealth**” or “**productive wealth**” for simplicity. Measured in prices of a base year (constant prices), the growth of wealth is a weighted average of the growth rates of the asset stocks. The weights are the shares of each asset in the value of wealth. Because the value of each asset is its price times its quantity, these weights are referred to as *asset price weights*.
- A second concept of aggregate capital is **capital services**. In this case, to obtain an aggregate measure of capital, different types of asset stocks are *weighted by their rental prices*.

What we, and the authors mentioned above, argue, is that capital services and not wealth should be used in a production function. In that sense, the MPK is the extra output obtained from a unit increase in the amount of capital services used by the firm. Of course, the two concepts of capital are related. The current price of a (productive) asset, in theory, should equal the present value of all the future streams of rental prices. However, as we show below, the correct price of capital to aggregate different types of assets is p^k , the price of capital, and not p^A , the price of the asset. This has implications for the evolution of wealth vs capital services over time. For instance, assets for which the rental price is high because they depreciate quickly or because its asset price falls faster, will have a higher weight in the capital services measure than in the wealth measure. This is the case, for instance, of IT assets whose weight in capital services will be higher than in wealth due to the above mentioned reasons.

A.2 The rental price and asset prices

Let us now define p^k and p^A . We follow very closely the example in [Oulton and Srinivasan \[2003\]](#) which in turn is based on the original contribution of [Jorgenson and Griliches \[1967\]](#). A leasing company buys a machine at $t - 1$ and leases it out at t . The price it pays to buy this machine of age 0, at time $t - 1$ is $p_{t-1,0}^A$. The value of this investment one period later will depend on the nominal rate of return r_t : $(1 + r_t)p_{t-1,0}^A$. This rate of return depends on various things. First, the leasing company is paid a rental $p_{t,0}^k$ during period t . This is the *rental price* for the services of capital (*not* the *asset price* p^A). By the end of period t the company has the same asset but now one year older with a market price of $p_{t,1}^A$. If we ignore taxes throughout for simplicity, the value of this investment is:

$$(1 + r_t)p_{t-1,0}^A = p_{t,0}^k + p_{t,1}^A. \quad (1)$$

If we now simply iterate this equation forward, assuming that the asset has a lifetime of N periods, we obtain the standard *asset pricing* formula: $p_{t-1,0}^A = \sum_{s=0}^N p_{t+s,s}^k / \prod_{\tau=0}^s (1 + r_{t+\tau})$.

Because we do not commonly observe rental prices as firms use their own capital, it is useful to derive the rental price from the asset prices which are observable in the data. From (1), we can obtain:

$$p_{t,0}^k = r_t p_{t-1,0}^A + (p_{t-1,0}^A - p_{t,1}^A) \quad (2)$$

The second term on the right-hand-side of (2) is the gain or loss from holding the asset between $t - 1$ and t . This has two components: depreciation and capital gain or loss. If we add and subtract $p_{t,0}^A$:

$$p_{t,0}^k = r_t p_{t-1,0}^A + (p_{t,0}^A - p_{t,1}^A) - (p_{t,0}^A - p_{t-1,0}^A) \quad (3)$$

The first term in brackets is depreciation (the difference today between the value of a new and old machine). The second term is the capital gain or loss from holding the same machine between two periods. If we assume a constant rate of depreciation $\delta = \frac{p_{t,0}^A - p_{t,1}^A}{p_{t,0}^A}$, then (3) becomes:

$$p_{t,0}^k = r_t p_{t-1,0}^A + \delta p_{t,0}^A - (p_{t,0}^A - p_{t-1,0}^A) \quad (4)$$

which is the Hall-Jorgenson (1967) formula for the cost of capital in discrete time.

So this expression gives us the *rental price* as a function of the rate of return, the depreciation rate, and the price of new assets. The prices of new assets are observable (otherwise we could not measure investment in constant prices). And since it depends only on prices of new assets, we can drop the age subscript. Also, since we are interested in measuring aggregate

capital, we need to consider that there are many assets indexed by i in the economy. Thus, we can write (4) as:

$$p_{it}^k = r_t p_{i,t-1}^A + \delta_i p_{it}^A - (p_{it}^A - p_{i,t-1}^A) \quad (5)$$

Notice that we are assuming here the same rate of return for all assets i consistent with profit maximization.¹ In practice, there may be frictions that may lead to different rates of returns for different assets. But since we are not modelling them here, we prefer to assume a common rate of return.

A.3 Aggregating over different assets

Let us ignore the problem of aggregating over different vintages of assets for simplicity and only address how to calculate a measure of aggregate capital in a production function when capital is made up of different assets. Since we assume in the model a neoclassical model of production and investment, here we simply assume a neoclassical model of capital aggregation. Suppose that the firm uses labor and several (M) types of assets to produce output:

$$Y_t = F(K_{1t}, K_{2t}, \dots, K_{Mt}, L_t, t) \quad (6)$$

We want to express output in terms of only labor and an aggregate capital measure:

$$Y_t = G(K_t, L_t, t) \quad (7)$$

How do we go from (6) to (7)? That is, how do we aggregate individual assets onto an aggregate measure K_t ? Take logs and totally differentiate (6) to obtain the rate of growth of output (growth rates denoted by a hat):

$$\hat{Y}_t = \sum_{i=1}^M \left(\frac{\partial \ln Y_t}{\partial \ln K_{it}} \right) \hat{K}_{it} + \frac{\partial \ln Y_t}{\partial \ln L_t} \hat{L}_t + \frac{\partial \ln Y_t}{\partial \ln t} \quad (8)$$

And using (7) we have:

$$\hat{Y}_t = \frac{\partial \ln Y_t}{\partial \ln K_t} \hat{K}_t + \frac{\partial \ln Y_t}{\partial \ln L_t} \hat{L}_t + \frac{\partial \ln Y_t}{\partial \ln t} \quad (9)$$

Equations (8) and (9) imply that:

$$\hat{K}_t = \sum_{i=1}^M \left[\left(\frac{\partial \ln Y_t}{\partial \ln K_{it}} \right) / \left(\frac{\partial \ln Y_t}{\partial \ln K_t} \right) \right] \hat{K}_{it} \quad (10)$$

¹For a *dollar* spent on capital at $t-1$ the rental price of capital formula is the familiar (ignoring taxes) expression $p_{it}^k = r_t + \delta_i + \Delta p_{it}^A (1 - \delta_i)$.

The derivatives in (10) are not directly observable, but if we make the neoclassical assumption that factors are paid their marginal revenue products, we would have that:

$$\frac{\partial \ln Y_t}{\partial \ln K_{it}} = \frac{p_{it}^k K_{it}}{p_t Y_t},$$

$$\frac{\partial \ln Y_t}{\partial \ln K_t} = \frac{p_t^k K_t}{p_t Y_t},$$

where p_t^k is the rental price of aggregate capital (the value of the marginal product of capital). Aggregate capital income is given by $p_t^k K_t = \sum_{i=1}^M p_{it}^k K_{it}$. This implies that:

$$\hat{K}_t = \sum_{i=1}^M \omega_{it} \hat{K}_{it}, \quad (11)$$

where

$$\omega_{it} = \frac{p_{it}^k K_{it}}{\sum_{i=1}^M p_{it}^k K_{it}} \quad (12)$$

What this equation suggests is that, to be consistent with the neoclassical production function, aggregate capital growth is the weighted sum of individual assets where the weights are determined by the income weights of the different assets. In practical terms, in discrete time, we use a Törnqvist chain index:

$$\ln[K_t/K_{t-1}] = \sum_{i=1}^M \bar{w}_{it} \ln[K_{it}/K_{i,t-1}], \quad \bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1}) \quad (13)$$

Note that these weights have used *rental prices* and **not** asset prices. Thus, aggregate capital must correspond to the definition of *capital services* given in section 1 above, and not with the definition of *wealth* since we are not using asset price weights. That is, aggregate capital in a production function is not the NIPA productive asset stocks, it is capital services. The BEA calculates wealth growth using the same formula, but where the weights are *asset price* weights (as it should for a wealth measure).² Calling W_t the level of productive capital, the growth of productive wealth is calculated by BEA as follows:

$$\ln[W_t/W_{t-1}] = \sum_{i=1}^M \bar{v}_{it} \ln[W_{it}/W_{i,t-1}], \quad \bar{v}_{it} = \frac{1}{2}(v_{it} + v_{i,t-1}), \quad (14)$$

²The PWT Tables also use a wealth measure of capital stocks. As Feenstra et al. [2015] state “Ideally, this would be a measure of capital services, not capital stocks. [...] However, the data requirements for estimating capital services are higher than for a capital stock measure”.

where now the weights used are the shares of each type of asset (W_{it}) in the nominal capital stock:

$$v_{it} = \frac{p_{it}^A W_{it}}{\sum_{i=1}^M p_{it}^A W_{it}} \quad (15)$$

Since $\sum_{i=1}^M p_{it}^A K_{it} = \sum_{i=1}^M p_{it}^A W_{it}$, the only difference between (13) and (14)-(15) is in the weights used. As mentioned above, these two measures will differ because capital services will give a higher weight to assets whose depreciation is high and whose asset prices are falling faster over time.

A.4 Real wealth and real capital services

To calculate real wealth, we simply take the level of nominal wealth in a specific (base) year s : $\sum_{i=1}^M p_{is}^A W_{is}$. Then we generate a series of, say, chained 2009 dollars by applying the rate of growth formula (14)-(15) to this 2009 value.

By the same token, we can calculate a series for the real level of capital stock by calculating the nominal level for a base period: $\sum_{i=1}^M p_{is}^k K_{is}$, and then applying the growth rate formula (13) to calculate the level of capital services at 2009 dollars. Notice that, in the base year, this amounts to setting capital services to total capital income. This is why in the BLS file “Historical Multifactor Productivity Measures (SIC 1948–87 linked to NAICS 1987–2017)”³ capital services coincide with capital income in the base period. This, obviously, does not imply that capital services equal capital income in each period.

B Intertemporal Problem

The representative household maximizes the discounted sum of utilities

$$\sum_{t=0}^{\infty} \beta^t V(p_{st}, p_{gt}, E_t).$$

where $V(p_s, p_g, E)$ is the indirect utility function

$$V(p_s, p_g, E) = \frac{1}{\epsilon} \left[\frac{E}{p_s} \right]^\epsilon - \frac{\nu}{\gamma} \left(\frac{p_s}{p_g} \right)^{-\gamma} - \frac{1}{\epsilon} + \frac{\nu}{\gamma}.$$

where $0 \leq \epsilon \leq \gamma \leq 1$ and $\nu > 0$.

³Available at <https://www.bls.gov/mfp/mprdload.htm#Historical%20Series>.

The budget constraint of the consumer is

$$E_t + K_{t+1} = w_t + K_t(1 + r_t - \delta)$$

Note that total expenditure is

$$E_t = p_{st}C_{st} + C_{gt}$$

In the intertemporal problem the household chooses the variables E_t and K_{t+1} . Due to the indirect utility function the intratemporal problem is already solved. In the intratemporal problem the household, given E_t chooses the variables C_{st} and C_{gt} .⁴

The Lagrangean of the household is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t V(p_{st}, p_{gt}, E_t) + \sum_{t=0}^{\infty} \mu_t [w_t + K_t(1 + r_t - \delta) - E_t - K_{t+1}].$$

The household maximizes with respect to E_t and K_{t+1} .

Let us compute first the derivative of V with respect to E_t . This is

$$V'_E(p_{st}, p_{gt}, E_t) = \left[\frac{1}{p_{st}} \right]^\epsilon E_t^{\epsilon-1} \quad (16)$$

Then FOCs are

$$\beta^t \left[\frac{1}{p_{st}} \right]^\epsilon E_t^{\epsilon-1} = \mu_t \quad (17)$$

$$\mu_{t+1}[1 + r_{t+1} - \delta] = \mu_t \quad (18)$$

From (17) and (18) we obtain

$$\beta[1 + r_{t+1} - \delta] = \left[\frac{p_{st+1}}{p_{st}} \right]^\epsilon \frac{E_t^{\epsilon-1}}{E_{t+1}^{\epsilon-1}} \quad (19)$$

and using the fact that

$$p_{st} = \frac{(A_{gt})^{1-\alpha}}{(A_{st})^{1-\alpha}}$$

we can write

$$\beta[1 + r_{t+1} - \delta] = \left[\left(\frac{A_{gt+1}}{A_{st+1}} \right)^{1-\alpha} \left(\frac{A_{st}}{A_{gt}} \right)^{1-\alpha} \right]^\epsilon \left(\frac{E_t}{E_{t+1}} \right)^{\epsilon-1} \quad (20)$$

which is the **Euler Equation**.

⁴Note that in writing the intertemporal and static budget constraints we are already assuming that p_{gt} is the numeraire each period.

If we define the growth factor of technology in each sector as

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s = g_s \quad (21)$$

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g = g_g \quad (22)$$

we can write

$$\beta[1 + r_{t+1} - \delta] = \left(\frac{g_g}{g_s}\right)^{(1-\alpha)\epsilon} \left(\frac{E_{t+1}}{E_t}\right)^{1-\epsilon} \quad (23)$$

By rewriting the Euler equation as

$$\frac{E_{t+1}}{E_t} = \{\beta[1 + r_{t+1} - \delta]\}^{\frac{1}{1-\epsilon}} \left(\frac{g_s}{g_g}\right)^{\frac{(1-\alpha)\epsilon}{1-\epsilon}}, \quad (24)$$

we can compute the *intertemporal elasticity of substitution of consumption expenditure* (EIS). The EIS is defined as the derivative of log total expenditure with respect to the log of the interest rate holding within-period relative prices and the discounted marginal utility of expenditure constant, where expenditure and the interest rate are nominal.⁵ We can thus define the EIS as (see [Blundell et al. \[1994\]](#))

$$EIS = \frac{\frac{d(E_{t+1}/E_t)}{E_{t+1}/E_t}}{\frac{d(1+r_{t+1}-\delta)}{1+r_{t+1}-\delta}} = -\frac{V'_E}{V''_{EE}E} = \frac{1}{1-\epsilon}. \quad (25)$$

C Balanced Growth

We now look for a balanced growth path. We assume that expenditure grows at a constant rate, that is $E_{t+1}/E_t = 1 + \gamma_E = g_E$ so from the Euler equation

$$\beta[1 + r_{t+1} - \delta] = \left(\frac{g_g}{g_s}\right)^{(1-\alpha)\epsilon} (1 + \gamma_E)^{1-\epsilon}$$

and

$$r_t = \frac{1}{\beta} \left(\frac{g_g}{g_s}\right)^{(1-\alpha)\epsilon} (g_E)^{1-\epsilon} - 1 + \delta. \quad (26)$$

⁵The alternative would be to relate expenditure deflated by a single price index to the real interest rate. As [Gorman \[1959\]](#) shows, however, deflation by a single price index requires within period homotheticity, which is not the case in this setting.

The FOCs of capital for the goods firm is

$$r_t = \alpha \left(\frac{k_{gt}}{n_{gt}} \right)^{\alpha-1} A_{gt}^{1-\alpha}$$

and, as that the capital/labor ratio in each sector is equal to the economy wide one in equilibrium, we can equate it to (26) to obtain

$$\frac{1}{\beta} \left(\frac{g_g}{g_s} \right)^{(1-\alpha)\epsilon} (1 + \gamma_E)^{1-\epsilon} - 1 + \delta = \alpha (K_t)^{\alpha-1} A_{gt}^{1-\alpha}$$

$$\frac{1}{\beta} \left(\frac{g_g}{g_s} \right)^{(1-\alpha)\epsilon} (g_E)^{1-\epsilon} - 1 + \delta = \alpha \left(\frac{A_{gt}}{K_t} \right)^{1-\alpha}.$$

For this equation to hold in each period it must be that $1 + \gamma_k = g_k = g_g$. So aggregate capital must grow at the same rate as the technology in the goods sector.

We can write total output in units of the numeraire (goods) as

$$Y_{gt} = K_t^\alpha A_{gt}^{1-\alpha}$$

It is then straightforward to show that Y_{gt} also grows at factor g_g . Also, wages are given by

We can write the wage as

$$w_t = (1 - \alpha) K_t^\alpha A_{gt}^{1-\alpha}$$

and so it also grows at factor g_g .

Finally, using the dynamic budget constraint

$$E_t + K_{t+1} = w_t + K_t(1 + r_t - \delta)$$

we observe that both K and w grow at rate γ_g and so it must hold that $\gamma_E = \gamma_g$, confirming the initial assumption that expenditure grows at a constant rate and so the existence of a balanced growth path.

D From Boppart (2014) to the ISTC model

The **indirect utility function** is obtained with $\nu = 0$ and it is given by equation (20) in the main text. Recall that we drop the terminology *services* and *goods* in this model to refer to *consumption* and *investment* sectors.

In the intertemporal problem the representative household maximizes the discounted sum

of utilities

$$\sum_{t=0}^{\infty} \beta^t V(p_t, E_t),$$

subject to the budget constraint

$$E_t + K_{t+1} = w_t + K_t(1 + r_t - \delta).$$

Note that the derivative of V with respect to E_t is the same as in the structural change model

$$V'_E(p_t, E_t) = \left[\frac{1}{p_t} \right]^\epsilon E_t^{\epsilon-1} \quad (27)$$

so the FOCs are also identical to the structural change model

$$\beta^t \left[\frac{1}{p_t} \right]^\epsilon E_t^{\epsilon-1} = \mu_t, \quad (28)$$

$$\mu_{t+1}[1 + r_{t+1} - \delta] = \mu_t, \quad (29)$$

implying that the Euler equation is also the same

$$\beta[1 + r_{t+1} - \delta] = \left[\left(\frac{A_{It+1}}{A_{ct+1}} \right)^{1-\alpha} \left(\frac{A_{ct}}{A_{It}} \right)^{1-\alpha} \right]^\epsilon \left(\frac{E_t}{E_{t+1}} \right)^{\epsilon-1}, \quad (30)$$

and we have used the definition of p_t given by equation (25) in the main text. It follows that all the characterization of the balanced growth path is the same as in the structural change model.

E Calibration of the ISTC model

From (30) we can isolate r_{t+1} and equate the resulting expression to the marginal product of capital of the investment firm to obtain

$$\frac{1}{\beta} \left(\frac{g_I}{g_c} \right)^{(1-\alpha)\epsilon} (g_E)^{1-\epsilon} - 1 + \delta = \alpha \left(\frac{A_{It}}{K_t} \right)^{1-\alpha},$$

where we used

$$\frac{A_{ct+1}}{A_{ct}} = 1 + \gamma_c = g_c, \quad (31)$$

$$\frac{A_{It+1}}{A_{It}} = 1 + \gamma_I = g_I, \quad (32)$$

$$\frac{E_{t+1}}{E_t} = 1 + \gamma_E = g_E, \quad (33)$$

and the fact that the capital/labor ratio in each sector is equal to the economy wide one. From the above we can find the value of initial capital consistent with balanced growth

$$K_0 = \frac{(\alpha\beta)^{1/(1-\alpha)} A_{I0}}{\left[\left(\frac{g_I}{g_c} \right)^{(1-\alpha)\epsilon} (g_E)^{1-\epsilon} - \beta(1+\delta) \right]^{1/(1-\alpha)}}$$

and using that $g_E = g_I$

$$K_0 = \left\{ \frac{\alpha\beta}{[g_I^{1-\alpha\epsilon} g_c^{\alpha\epsilon-\epsilon} - \beta(1+\delta)]} \right\}^{1/(1-\alpha)} A_{I0}$$

and

$$r_t = \frac{1}{\beta} g_I^{1-\alpha\epsilon} g_c^{\alpha\epsilon-\epsilon} - 1 + \delta. \quad (34)$$

We can now write the nominal investment rate in the initial period $\frac{I_0}{Y_{I0}}$, where Y_{I0} is output in investment units in the initial period.

$$\begin{aligned} \frac{I_0}{Y_{I0}} &= \frac{K_1 - (1-\delta)K_0}{(K_0)^\alpha A_{I0}^{1-\alpha}} \\ \frac{I_0}{Y_{I0}} &= \frac{(1+\gamma_I)K_0 - (1-\delta)K_0}{(K_0)^\alpha A_{I0}^{1-\alpha}} \\ \frac{I_0}{Y_{I0}} &= \frac{(\gamma_I + \delta)K_0}{(K_0)^\alpha A_{I0}^{1-\alpha}} \\ \frac{I_0}{Y_{I0}} &= \frac{(\gamma_I + \delta)}{A_{I0}^{1-\alpha}} K_0^{(1-\alpha)} \end{aligned}$$

and using K_0

$$\begin{aligned} \frac{I_0}{Y_{I0}} &= \frac{(\gamma_I + \delta)}{A_{I0}^{1-\alpha}} \left\{ \frac{\alpha\beta}{[g_I^{1-\alpha\epsilon} g_c^{\alpha\epsilon-\epsilon} - \beta(1+\delta)]} \right\} A_{I0}^{1-\alpha} \\ \frac{I_0}{Y_{I0}} &= \frac{\alpha\beta(\gamma_I + \delta)}{[g_I^{1-\alpha\epsilon} g_c^{\alpha\epsilon-\epsilon} - \beta(1+\delta)]} \end{aligned} \quad (35)$$

and so the nominal investment rate is constant in the first period and in all subsequent ones.

Define $I_0/Y_{I0} = s$, $C_0 p_0/Y_{I0} = 1 - s$, where $C_0 = E_0/p_0$. Also, as consumption at each t

is given by

$$C_t = \frac{E_t}{p_t} = \frac{E_t}{(A_{It})^{1-\alpha}} (A_{ct})^{1-\alpha},$$

we have that the growth rate of consumption along the balanced growth path is

$$\gamma_{Cons} = \gamma_E + (1 - \alpha) \gamma_c - (1 - \alpha) \gamma_I$$

$$\gamma_{Cons} = (1 - \alpha) \gamma_c + \alpha \gamma_I.$$

The Laspeyres chain-weighted index of GDP is

$$Q_t^L = \frac{I_1 + p_0 C_1}{I_0 + p_0 C_0},$$

and using model's growth rates along the balanced growth path

$$\begin{aligned} Q_t^L &= \frac{I_0(1 + \gamma_I) + p_0 C_0 [1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}{I_0 + p_0 C_0}, \\ Q_t^L &= \frac{s Y_{I0}(1 + \gamma_I) + (1 - s) Y_{I0} [1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}{Y_{I0}}, \\ Q_t^L &= s(1 + \gamma_I) + (1 - s) [1 + (1 - \alpha) \gamma_c + \alpha \gamma_I], \\ Q_t^L &= 1 + s \gamma_I + (1 - s) [(1 - \alpha) \gamma_c + \alpha \gamma_I]. \end{aligned}$$

So the Laspeyres growth factor is a constant weight average of real investment and consumption growth.

The Paasche chain-weighted index of GDP is

$$\begin{aligned} Q_t^P &= \frac{I_1 + p_1 C_1}{I_0 + p_1 C_0}, \\ Q_t^P &= \frac{I_1 + p_1 C_1}{I_1/(1 + \gamma_I) + p_1 C_1/[1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}, \\ Q_t^P &= \frac{Y_{I1}}{s Y_{I1}/(1 + \gamma_I) + (1 - s) Y_{I1}/[1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}, \\ Q_t^P &= \frac{1}{s/(1 + \gamma_I) + (1 - s)/[1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}. \end{aligned}$$

The Fisher chain-weighted index of GDP is then

$$Q_t^F = \sqrt{Q_t^L Q_t^P} = \sqrt{\frac{s(1 + \gamma_I) + (1 - s) [1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}{s/(1 + \gamma_I) + (1 - s)/[1 + (1 - \alpha) \gamma_c + \alpha \gamma_I]}}. \quad (36)$$

So GDP growth, as measured with a Fisher index, is constant in the ISTC model. As both nominal GDP (i.e. GDP in investment units) and real GDP grow at a constant rate, also the relative price investment/GDP grows (declines) at a constant rate.

In the calibration in the paper we equate (35), (36) and the growth rate of I/Y to the data counterparts to pin down γ_I , γ_c and ϵ .

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