Heterogeneous Productivity Shocks, Elasticity of Substitution and Aggregate Fluctuations *

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Abstract

We use a Dixit-Stiglitz setting to show that aggregate productivity fluctuations can be generated through changes in the dispersion of firms' productivity. When the elasticity of substitution among goods is larger than one, an increase in the dispersion raises aggregate productivity because firms at the top of the distribution produce most of output. When the elasticity is smaller than one, an increase in the dispersion reduces aggregate productivity because firms at the bottom of the distribution use most of inputs. We use individual firm level data from Spanish manufacturing firms to test the relationship between the dispersion of firms' productivity and aggregate productivity. The estimated coefficients are consistent with the predictions of the model: we find that an increase in the coefficient of variation of firms productivity of 1% increases aggregate productivity by 0.16% in sectors with an elasticity of substitution larger than one while the same increase in the standard deviation reduces aggregate productivity by 0.36% in sectors with an elasticity of substitution smaller than one.

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1 Introduction

In this paper we study the relationship between a time varying distribution of firms idiosyncratic productivity and aggregate productivity fluctuations. We first use a simple general equilibrium model with Dixit-Stiglitz indices to show that when the elasticity of substitution among a large number of goods is different from one, aggregate productivity is different from the average productivity of firms producing those goods. This implies that even if the deterministic part of firms productivity is equal to one and firms receive i.i.d. shocks from a common probability distribution function, aggregate productivity is different from one.

This result follows from the fact that the elasticity of substitution determines consumers' willingness to change the purchases ratio of two goods when the price ratio of those goods changes. If the elasticity is high, consumers switch from one good to another for small price changes. When the elasticity is small, it takes high price differentials to induce consumers to slightly change the bundle of goods they are consuming. Thus, a low elasticity of substitution implies that production is distributed evenly across producers. If this is the case, low productivity firms have a large impact on the productive capacity of the economy and therefore aggregate productivity is low. On the other hand, when the elasticity of substitution is high, output is produced mostly by high productivity firms and aggregate productivity is large. Put it differently, the share of more productive firms in industry revenue increases with the degree of substitutability of products. It follows that, since aggregate productivity can be seen as an output-weighted average of firms productivity, it increases with the elasticity of substitution among products.

An implication of an elasticity of substitution different from one is that, even when the mean of the distribution does not change, changes in the shape of the distribution of firms' productivity have the same effect of an aggregate shock hitting the productivity of all firms. Although this paper does not provide a theory of the time variation of the distribution of firms' productivity, there is little reason to suppose that this distribution is stable over time. On the empirical side, Bloom et al. (2012) provide evidence suggesting that the variance of establishment, firm and industry level shocks in the U.S. is countercyclical. Kehrig (2011) shows that the dispersion of the level of firms productivity in U.S. manufacturing is counter-cyclical and it is more pronounced in durables than in non-durables. Bachman and Bayer

(2009), using a panel of public and private German firms in manufacturing and retail, find that the variance of innovations to firms' productivity increases in recessions. In this paper, we provide evidence that the variance of the distribution of firms productivity in Spanish manufacturing sectors varies sensibly over time. It follows that the interaction between a time varying productivity distribution and an elasticity of substitution different from one provides a source of fluctuations in aggregate productivity without the need to assume a common (aggregate) shock to the productivity of all firms, or an input-output matrix that transmits sectoral shocks across sectors.

A crucial point here is that, depending on the elasticity of substitution, an increase in the dispersion of firms' productivity can have either a positive or a negative effect on aggregate productivity. When the elasticity is smaller than one, an increase in the dispersion has a negative effect, while the opposite holds with an elasticity larger than one. This happens because an increase in the dispersion implies that there are more high productive firms and more low productive firms. If the elasticity of substitution is high, most productive firms employ most of inputs and produce most of output so when their number increases aggregate productivity also increases. When the elasticity of substitution is low, demand tends to be distributed evenly among producers, so an increase in the number of low productive firms reduces aggregate productivity because these firms use most of inputs.

To test the predictions of the model we use data from 18 Spanish manufacturing sectors. We first estimate the elasticity of substitution among goods in each sector. This is smaller than one in 14 sectors and larger than one in 4 sectors. With the estimated elasticity of substitution we are able to construct, for each sector, the relevant measure of aggregate productivity. According to the model, sectors with an elasticity of substitution larger (lower) than one show an increase (decrease) in aggregate productivity when the dispersion of the productivity distribution increases. We test this implication in a regression framework. We regress aggregate productivity of each sector on the coefficient of variation of productivity in each sector and the interaction between the coefficient of variation and a dummy variable that takes value one if the sector has an elasticity of substitution larger than one. The estimated coefficients are consistent with the predictions of the model; we find that an increase of 1% in the coefficient of variation of the distribution of firms' productivity increases aggregate productivity by 0.16% in sectors with an elasticity of substitution larger than one while the same increase in the coefficient of variation reduces aggregate productivity by 0.36% in sectors with an elasticity of substitution smaller than one.

We are not the first to investigate the effects of a time varying dispersion of firms' productivity on aggregate fluctuations. The already mentioned papers by Bachman and Bayer (2009), Bloom et al. (2012) and Kehrig (2011) provide fully-fledged general equilibrium models that allow to study these effects. Bloom et al. (2012) show that when labor and capital adjustment costs are present, uncertainty shocks make firms more cautious, thus delaying hiring and investment, which in turn depresses aggregate productivity and economic activity. Bachman and Bayer (2009) instead, stress the "news" role of changes in uncertainty in shaping aggregate fluctuations. Kehrig (2011) presents a model with overhead inputs–that become more expensive in booms–and entry and exit of firms. In equilibrium, only the most productive new firms enter and only the most productive incumbents survive during economic expansions.[1] Compared to these contributions, we identify a new channel through which changes in the dispersion of firms' productivity can lead to aggregate fluctuations. This is solely grounded in the elasticity of substitution among goods.[2]

This paper also contributes to two other strands of the literature: the one that studies the existence of persistent productivity differences among firms and the one that studies how the distribution of resources among firms affects aggregate productivity. Within the former, a closely related paper is Syverson (2004), who investigates the role of the elasticity of substitution on observed differences in plant level productivity. He points out, focusing on the concrete market, that barriers to substitutability of any kind (spatial, physical or brand driven) among producers, allow less productive firms to survive, thus decreasing average productivity. Compared to Syverson (2004) we focus on the effect of goods substitutability on aggregate fluctuations.

In the increasing literature on the distribution of resources across firms

[1]See also Heathcote, Storesletten and Violante (2014), who present a model in which the dispersion of wages across individuals is time varying.

[2]More broadly, our paper also relates to the literature on the ability of models with a large number of sectoral shocks to generate aggregate fluctuations. Lucas (1981) and Dupor (1999) suggest that when the economy is sufficiently disaggregated, independent sectoral shocks wash out in the aggregate because of the law of large numbers. Instead, Horvath (1998), and in particular Acemoglu et al. (2012), show that the response of the aggregate economy to a large number of sectoral shocks depends on the input-output structure of the economy.

and aggregate productivity, Restuccia and Rogerson (2008) show how a misallocation of resources reduces aggregate TFP; Guner et al. (2008) analyze the role of restrictions on the size of firms for aggregate productivity; Hsie and Klenow (2009) provide a quantitative evaluation of the impact of misallocation on aggregate TFP. In contrast with these contributions, we focus on an economy with no distortions. We show that changes in the distribution of resources across firms can have different effects on aggregate productivity depending on the elasticity of substitution among goods.

The remaining of the paper is as follows: section 2 describes the model; section 3 reports the quantitative results; section 4 presents some robustness checks; and finally, section 5 concludes.

2 The Model

2.1 Sectors and Firms

We consider an economy with $i = 1, 2, .., n$ *broad sectors*, each producing a good also indexed by i. In turn, each broad sector i is composed of a set of *atomless sectors* indexed by $j \in [0, 1]$. In each sector j there is perfect competition and the *representative firm* in j produces output using the following production function

$$y_{ij} = A_{ij} N_{ij}, \tag{1}$$

where N_{ij} is the amount of labor used in production and A_{ij} is a firm specific productivity term. It follows that the representative firm j maximizes profits according to the zero profit condition

$$p_{ij} = \frac{w}{A_{ij}}, \tag{2}$$

where w is the wage rate and p_{ij} the price of output.

2.2 Household

There is a representative household in the economy consuming all goods from the $1, 2, .., n$ broad sectors. The utility function is

$$U = \Sigma_{i=1}^{n} \alpha_i \log C_i, \tag{3}$$

with $\Sigma_{i=1}^n \alpha_i = 1$. Consumption from sector i is a Dixit-Stiglitz aggregator of the goods purchased in that sector

$$C_i = \left[\int_0^1 c_{ij}^{\frac{\theta_i - 1}{\theta_i}} dj \right]^{\frac{\theta_i}{\theta_i - 1}}. \tag{4}$$

The household is endowed with one unit of labor that she supplies inelastically in the market, earning the wage w. Thus, her budget constraint is

$$\Sigma_{i=1}^n P_i C_i = w, \tag{5}$$

where P_i is the price index associated to the consumption index C_i. The problem of the household is then to maximize (3) subject to (4) and (5).

2.3 Equilibrium

A competitive equilibrium for this economy is a wage rate w and, for each sector i, a set of prices $\{p_{ij}\}_{j\in[0,1]}$, a set of allocations $\{c_{ij}\}_{j\in[0,1]}$ for the household, a set of allocations $\{y_{ij}, N_{ij}\}_{j\in[0,1]}$ for the representative firms, a consumption index C_i and a price index P_i such that:

a) given prices and the wage rate, allocations $\{c_{ij}\}_{j\in[0,1]}$ of sectors $i = 1, .., n$ solve the household's problem;

b) given prices and the wage rate, allocations $\{y_{ij}, N_{ij}\}$ solve the problem of the representative firm ij;

c) the price and consumption indices of sector i are such that $P_i C_i = \int_0^1 p_{ij} c_{ij} dj$;

d) goods and labor markets clear:

$$y_{ij} = c_{ij} \quad \forall i, j \tag{6}$$

$$\sum_{i=1}^n N_i = \sum_{i=1}^n \int_0^1 N_{ij} dj = 1 \tag{7}$$

2.4 Model Solution

To solve the model, it is convenient to split the household's problem in two stages. First, given price indices P_i, the consumer maximizes (3) subject to (5). The solution to this problem gives

$$C_i = \frac{\alpha_i w}{P_i}. \tag{8}$$

Second, as (8) gives the amount of resources the household optimally spends in sector i, it is possible to solve for the optimal demand of goods within sector i

$$\max_{c_{ij}} C_i = \max_{c_{ij}} \left[\int_0^1 c_{ij}^{\frac{\theta_i - 1}{\theta_i}} dj \right]^{\frac{\theta_i}{\theta_i - 1}} \tag{9}$$

subject to

$$\int_0^1 p_{ij} c_{ij} dj = w\alpha_i,$$

where to derive the constraint in problem (9) we assumed $P_i C_i = \int_0^1 p_{ij} c_{ij} dj$. Below we prove that this condition holds in equilibrium. The first order conditions for problem (9) deliver the demand function for firm j in sector i:

$$c_{ij} = \left(\frac{p_{ij}}{P_i} \right)^{-\theta_i} C_i. \tag{10}$$

Also, the demand function (10) implies that the price of C_i is

$$P_i = \left[\int_0^1 p_{ij}^{1-\theta_i} dj \right]^{\frac{1}{1-\theta_i}}. \tag{11}$$

Note that using (10) and (11) it can be proved that $P_i C_i = \int_0^1 p_{ij} c_{ij} dj$. Finally, from (1) and (2) it holds that $p_{ij} y_{ij} = w N_{ij}$. Integrating between 0 and 1 and noting that the total amount of labor used in sector i is $N_i = \int_0^1 N_{ij} dj$, we can write

$$\int_0^1 p_{ij} c_{ij} dj = w N_i. \tag{12}$$

By comparing (12) and the constraint of problem (9) it follows that the amount of labor used in sector i is

$$N_i = \alpha_i.$$

2.5 Aggregate and Average productivity

By using (2) to substitute for p_{ij} in (11) it follows that

$$\frac{w}{P_i} = \left[\int_0^1 A_{ij}^{\theta_i - 1} dj \right]^{\frac{1}{\theta_i - 1}}. \tag{13}$$

Next, by using (13) in (12) and $P_i C_i = \int_0^1 p_{ij} c_{ij} dj$, we obtain

$$C_i = \left[\int_0^1 A_{ij}^{\theta_i - 1} dj \right]^{\frac{1}{\theta_i - 1}} N_i. \tag{14}$$

Equation (14) represents the aggregate production function of sector i, as it maps the total amount of labor used in production in that sector into total output. In (14), the productivity term $\left[\int_0^1 A_{ij}^{\theta_i - 1} dj \right]^{\frac{1}{\theta_i - 1}}$ depends on individual firms productivity A_{ij}, and on the elasticity of substitution θ_i among goods produced in sector i.[3] Note that

$$A_i = \left[\int_0^1 A_{ij}^{\theta_i - 1} dj \right]^{\frac{1}{\theta_i - 1}}, \tag{15}$$

represents *aggregate productivity* of sector i and differs from *average productivity* $\bar{A}_i = \int_0^1 A_{ij} dj$ within the sector. Thus, depending on the elasticity of substitution among goods, the distribution of firms' individual productivity A_{ij} implies different levels of aggregate productivity.[4]

2.6 Implications

Equation (15) shows that aggregate productivity is a geometric, and not a linear, mean of individual productivity. Thus, when $\theta_i \neq 1$, the linear aggregation of firms' productivity does not provide an appropriate measure of aggregate productivity. To see the importance of θ_i in shaping aggregate productivity, assume that the productivity of firm j in i is $A_{ij} = e^{\varepsilon_j}$, where each ε_j is an i.i.d. shock from a $N(0, \sigma^2)$ distribution. Thus, if the shock ε_j is zero, the productivity of firm j is equal to one. Then, by applying Theorem 2 in Uhlig (1995), it can be shown that

$$A_i = \left[\int_0^1 A_{ij}^{\theta_i - 1} dj \right]^{\frac{1}{\theta_i - 1}} \equiv e^{(\theta_i - 1)\sigma^2/2}, \tag{16}$$

[3] We are assuming that labor is the only input in production. Therefore aggregate labor productivity is equal to aggregate TFP.

[4] Note that, as we abstract from capital in the model, the individual firm productivity term can be interpreted as including the amount of capital used by the same firm. Thus, the demand elasticity of substitution can be also interpreted as a production elasticity of substitution in (14). In this view, our results are related to De La Grandville (2009), who shows that in an aggregate model with capital and labor inputs, the larger this elasticity of substitution, the faster the growth rate of the economy.

with probability one. As equation (16) makes clear, the value of the elasticity of substitution θ_i determines the effect that a change in the dispersion of firms' productivity has on aggregate productivity. With a unitary elasticity of substitution, $\theta_i = 1$, changes in the dispersion of firms' productivity have no effect on aggregate productivity. When $\theta_i > 1$ most productive firms produce a large part of output in the economy. When the variance of the productivity distribution increases, the number of firms in the tails of the distribution increases, implying that there are more high productive, and more low productive firms. As output is produced mainly by high productive firms when $\theta_i > 1$, these firms attract most of labor, and an increase in the variance raises aggregate productivity. When $\theta_i < 1$ production tends to be divided evenly among producers in the economy. To see why an increase in the variance of the productivity distribution reduces aggregate productivity when $\theta_i < 1$, consider the limit case in which θ_i tends to zero (Leontief demand). In this case, production is divided equally among producers, regardless of prices. In this situation, the least productive firm determines the amount of output demanded to (and thus produced by) all firms. When the variance of the productivity distribution increases the number of firms at the bottom of the distribution increases. As the elasticity of substitution is small, these firms attract most of the labor input in the economy and aggregate productivity declines.

Summarizing, the main prediction of the model is that a time-varying dispersion in firms' productivity induces aggregate productivity fluctuations when the elasticity of substitution among goods is different from one. When the elasticity of substitution is different from one, changes in aggregate productivity can be the result of changes in the shape of the productivity distribution (in the example above a change in the variance of the distribution) instead of changes in the productivity of each firm (common aggregate shocks). In the next section we investigate whether the interaction between changes in the distribution of productivity shocks and an elasticity of substitution different from one has a quantitatively relevant impact on aggregate productivity fluctuations.

3 Quantitative Analysis

3.1 Data and variables

We use individual firm-level data from the *Survey on Business Strategies* (*Encuesta sobre Estrategias Empresariales*, ESEE) which is an annual survey of a representative sample of Spanish manufacturing firms. The sample covers the period 1991-2005. In the first year, firms were chosen according to a sampling scheme where weights depend on size. All firms with more than 200 employees were surveyed and their participation rate in the survey reached approximately 70% of the overall population of firms in this category. Likewise, firms with 10 to 200 employees were surveyed according to a random sampling scheme with a participation rate close to 5%. This selection scheme was applied to each industry in the manufacturing sector. Another important feature of the survey is that the initial sample properties have been maintained in all subsequent years. Newly created and exiting firms have been recorded in each year with the same sampling criteria as in the base year. Therefore, due to this entry and exit process, the dataset is an unbalanced panel of firms. The number of firms with information on all the variables of interest is 3,277 and the number of observations is 18,247.[5]

We classified firms in 18 industries according NACE classification. We first estimate the elasticity of substitution θ_i for each one of the 18 sectors ($i = 1, 2, \dots, 18$). Note that for each sector i, the demand function of firm j in the model, equation (10), can be written in logs as $\log c_{ijt} = -\theta_i \log p_{ijt} + \theta_i \log P_{it} + \log C_{it}$. To obtain an estimating equation at the firm level we replace $\theta_i \log P_{it} + \log C_{it}$ by an industry specific set of time dummy variables, η_{it}. By doing this we control for every non-observed time varying factor that affects homogeneously all firms in the same sector. Additionally, we include a non-observed time invariant term, μ_{ij}, that captures firm specific characteristics. Finally, we include a random term v_{ijt} that captures innovations that are not correlated with p_{ijt}. Therefore, for each sector i we estimate

$$\log c_{ijt} = -\theta_i \log p_{ijt} + \eta_{it} + \mu_{ij} + v_{ijt}, \quad i = 1, 2, \dots, 18, \tag{17}$$

where c_{ijt} is output of firm j in period t and p_{ijt} its price.[6] The output measure we use is the value of production in period t deflated using a firm specific price index (i.e. gross output). The price index is the same we use as regressor and is constructed as a Paasche-type price index computed from the percentage price changes that firms report to have made in the markets in which they operate. Because of the unobserved fixed effect, μ_{ij},

[5] The number of firms with 1, 2, 3, ..., 15 observations is 899, 359, 239, 190, 195, 221, 136, 127, 170, 120, 122, 103, 116, 133, 147, respectively. See Doraszelski and Jaumandreu (2013) and Escribano and Stucchi (2014) for additional details on the dataset.

[6] As in the model, subindex ij means that firm j belongs to sector i. In the dataset, firms never move from one sector to another.

we estimate (17) using the within group estimator.

Table 1 reports the estimation of θ_i for each industry i and its standard deviation. We find that all the coefficients but the one of "Other Manufactured Products" are positive, possibly due to the heterogeneity of products in that sector. There are two sectors with θ_i statistically larger than one, eight sectors with θ_i statistically lower than one, and for the remaining seven sectors it is not possible to reject the null hypothesis of $\theta = 1$.[7]

[Table 1 here]

Next, we look at firms productivity in each sector for each year of the sample period 1991-2005. We construct labor productivity by considering output per hour worked. Output is measured as described above. Table 2 reports the mean and standard deviation over time of the two main variables that we use in the analysis in the next section–i.e. the sector level of productivity and the sector coefficient of variation of productivity. That is, for each sector and each year, we first compute the mean and the coefficient of variation of firms productivity. Then, we compute the mean over time and the standard deviation over time of these two measures.

[Table 2 here]

3.2 Testing the implications of the model

Tables 1 and 2 provide empirical evidence about the two basic ingredients of our model: (a) the elasticity of substitution among goods is different from one in all sectors, ranging from 0.47 to 2.49; and (b) the dispersion of firms productivity shows a certain degree of time variation in all sectors. Our model suggests that in sectors in which the elasticity of substitution among goods is larger than one, an increase in the dispersion of firms' productivity increases aggregate productivity, while the opposite holds when the elasticity of substitution is smaller than one.

[7] Broda and Weinstein (2006) estimate elasticities of susbsitution for products at different aggregation levels—3, 5 and 7 digit SITC—using US imports data for the period 1972-2001, finding an average elasticity above one. As discussed in Broda and Weinstein (2006), the more disaggregated is the definition of industries, the more substitutable are goods and the larger is the estimated elasticity of substitution. In this paper we use a 2-digit industry classification level and find that the elasticity of substitution is smaller than one in several sectors.

This prediction can be formally tested by estimating the following regression

$$\begin{aligned} \log A_{it} &= \beta_1 \log CV_{it} + \beta_2 \log CV_{it} \times 1[\theta_i > 1] + \\ &+ \beta_3 \log \bar{A}_{it} + \rho_t + \phi_i + u_{it} \end{aligned} \tag{18}$$

where A_{it} is the aggregate level of productivity of sector i in period t, constructed using the discrete counterpart of (15) and the value of θ_i reported in Table 1, CV_{it} is the coefficient of variation of firms' productivity in sector i and period t, and $1[\cdot]$ is an indicator function—i.e., $1[\theta_i > 1]$ is a dummy variable that takes value one if θ_i is larger than one.[8] We control for the average productivity of industry i at time t, $\bar{A}_{it}$. This is important because we expect a positive relationship between the average and aggregate productivity in each industry. In (18) we do not allow $\bar{A}_{it}$ to vary with θ because the elasticity of substitution does not influence the transmission of average productivity to aggregate productivity. In the robustness checks in section 4 we drop this assumption. Finally, ρ_t represents unobserved factors that affect in the same way the productivity of all sectors (for instance an economy wide trend in productivity) and ϕ_i is a set of time-invariant unobserved characteristics of each sector.

Regarding the parameters of equation (18), the model implies $\beta_1 < 0$, $\beta_2 > 0$, and $|\beta_2| > |\beta_1|$. Table 3 shows the fixed-effect estimates of equation (18). Both the sign and the magnitude of the estimated coefficients are in line with the theoretical model, suggesting that the main predictions are supported by the data even after controlling for unobserved sectors fixed-effects and other factors that affect in the same way the productivity of all sectors. As expected, average productivity affects linearly aggregate productivity.

[Table 3 here]

From a quantitative perspective, the estimated β_1 and $\beta_1 + \beta_2$ represent the elasticity of aggregate productivity to the coefficient of variation when θ_i is smaller and larger than one, respectively. Results in Table 3 suggest that the response of aggregate productivity is positive when $\theta_i > 1$, while it

[8]The discrete counterpart of (15) for a number N of representative firms j in sector i is $A_i = \left[\sum_{j=1}^{N} \frac{A_{ij}^{\theta_i - 1}}{N}\right]^{\frac{1}{\theta_i - 1}}$.

is negative when $\theta_i < 1$. In the first case, an increase in the coefficient of variation of 1% increases aggregate productivity by 0.157%. In the second case, the same increase in the coefficient of variation reduces aggregate productivity by 0.364%. The fit of the model is good; the R-squared is 0.98.[9] Therefore, our results suggest that commonly measured aggregate productivity fluctuations can be in part the results of a time-varying dispersion of firms' productivity.

4 Robustness checks

In this section we present five robustness exercises. First, we estimate equation (18) considering only those sectors for which the estimation of θ is statistically significant different from one. Column [1] in Table 4 shows the results. We excluded seven sectors in which it is not possible to reject the null hypothesis that θ is equal to one.[10] We also exclude the sector with a negative value of θ because it has no economic interpretation. The sign, magnitude, and statistical significance of the estimated coefficients is very close to those of Table 3.

[Table 4 here]

Second, we allow the effect of the average productivity to vary by industries with θ larger and lower than one. Column [2] in Table 4 shows that, although the coefficient is significant, the results regarding dispersion and aggregate productivity are robust. In addition, the fixed-effect in Table 3 controls for differences in the capital labor ratio across industries that remain constant over time. However, if this ratio varies across time, the estimates are inconsistent. To control for this source of inconsistency, column [3] in Table 4 includes the capital labor ratio as additional control. As in the previous case, there are no significant changes in the sign, magnitude, and statistical significance of the estimated coefficients.

[9] This R-squared is 0.05 larger than that of a model that includes only the average productivity.

[10] These sectors are: *Chemical Products*, *Metallic Products*, *Other Transport Material*, *Beverages*, *Leather products and shoes*, *Wood and Furniture*, and *Plastic Products and Rubber*.

Third, the dispersion in firms' productivity could be affected by the entry and exit of firms from the market. To account for this possibility, we restrict our attention to the balanced panel and therefore we consider only those firms that have information for the 15 years of the sample period.[11] Column [4] in Table 4 shows that estimation results are similar to those obtained in Table 3. However, in this case, it is not possible to reject the hypothesis that when θ is larger than one, aggregate productivity is unaffected by the dispersion of firms productivity.[12]

Fourth, we estimated the demand equation (17) assuming that that price is exogenous. In our model the price p_{ij} does not depend on the quantity y_{ij} and therefore the assumption is valid. Here we relax this assumption and consider the possibility of an endogenous price. To solve the endogeneity problem, it is necessary to estimate equation (17) using instrumental variables. The endogeneity comes from the fact that we observe equilibrium prices and quantities that are also affected by the supply. Therefore, to identify the demand it is necessary to use a supply shifter. Our dataset provides us with information to construct a firm-level price index of intermediate goods purchased by each firm and therefore with the supply shifter needed.[13] The last column in Table 4 shows that the results are robust to the IV estimates of θ.

Finally, note that in the log-normal distribution the mean depends on the variance. That is, an increase in the dispersion of the distribution increases the mean. Our main theoretical result does not rest on the type of distribution assumed, but the cutoff point can be sensitive to this choice. To

[11] There are 147 firms reporting information during the entire sample period.

[12] Kehrig (2011) shows that entry and exit of firms is an important determinant of changes in productivity dispersion. By considering the balanced panel we are removing such source of fluctuations in our measure of aggregate productivity.

[13] The firm-level price index of intermediate goods is constructed as a Paasche-type price index computed from the percentage price changes that firms report to have paid in the intermediate materials they used. The price of intermediate goods resulted significant in the first stage regressions in 15 out of 18 sectors. As expected, an increase in the price of intermediate goods is translated into an increase in the price of the final good. We didn't estimated θ in the three sectors in which the price of intermediate goods is not a good instrument for the firm's output price. The estimate of θ resulted negative in three sectors (*Non Metallic Products*, *Vehicles and Motors*, and *Other Manufactured Products*); these negative values have no economic interpretation. Six of the remaining sectors display an elasticity larger than one and two an elasticity smaller than one. There are four sectors for which it is not possible to reject the null hypothesis of θ equal to one (*Metallic Products*, *Agricultural and Industrial Machinery*, *Food and Tobacco*, and *Wood and Furniture*).

see this, assume that that $logA_{ij} \sim N(-\frac{1}{2}\sigma^2, \sigma^2)$. In this case, $E(A_{ij}) = 1$, and aggregate productivity becomes $A_i = e^{(\theta_i - 2)\sigma^2/2}$. Clearly, also in this case aggregate productivity depends on the variance of the distribution and the elasticity of substitution, but the cutoff point becomes 2. To address this issue we estimate the model using the level of θ instead of a dummy variable. The estimation equation is:

$$\begin{aligned} \log A_{it} &= b_1 \log CV_{it} + b_2 \log CV_{it} \times \theta_i + \\ &+ b_3 \log \bar{A}_{it} + \rho_t + \phi_i + u_{it} \end{aligned} \tag{19}$$

In this way the cutoff point (if any) is selected by the estimation. The marginal effect of $\log A_{it}$ with respect to $\log CV_{it}$ is $b_1 + b_2 \times \theta_i$. Therefore, the cutoff point is given by $-b_1/b_2$. Results are presented in Table 5. The increase in dispersion reduces aggregate productivity when θ is lower than 1.2 and increases it when θ is larger than 1.8. Although the marginal effect is zero for θ between 1.2 and 1.8—a value somewhat larger that the cutoff point assumed in our benchmark estimation—this finding also suggests that for large values of θ an increase in the dispersion leads to an increase in aggregate productivity and the opposite holds for small values of θ.

[Table 5 here]

5 Conclusions

We presented a new mechanism that can generate fluctuations in aggregate productivity. This is given by the interaction between a time-varying dispersion of firms' productivity and an elasticity of substitution among goods different from one. The empirical evidence we provide suggests that the elasticity of substitution is different from one in manufacturing sectors, and the dispersion of firms' productivity varies sensibly over time. Thus, the mechanism we propose appears to have empirical relevance.

We also showe that, depending on the elasticity of substitution, the dispersion of firms' productivity can be positively or negatively related to the level of aggregate productivity. This suggests that the co-existence of firms with heterogeneous productivity is not necessarily detrimental for aggregate productivity: if the elasticity of substitution is high, the co-existence of high and low productive firms leads to higher aggregate productivity. This is not the case when the elasticity of substitution is low. In this case, the more homogeneous is the productivity across firms, the higher is the level of aggregate productivity.

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Table 1: Estimation of θ

	Sector	Coef.	sd
1	Ferric and Non Ferric Metals	1.34	0.09
2	Non Metallic Products	0.48	0.09
3	Chemical Products	0.99	0.11
4	Metallic Products	0.96	0.11
5	Agricultural and Industrial Machinery	0.72	0.14
6	Office Machinery, Data Processing Machinery	2.49	0.29
7	Electrical Material and Electrical Accessories	0.81	0.11
8	Vehicles and Motors	0.51	0.20
9	Other Transport Material	1.52	0.44
10	Meat and Meat Products	0.49	0.11
11	Food and Tobacco	0.47	0.08
12	Beverages	0.92	0.21
13	Textiles and Apparels	0.59	0.10
14	Leather products and shoes	0.58	0.28
15	Wood and Furniture	0.80	0.15
16	Paper, Paper Products and Printing Products	0.64	0.06
17	Plastic Products and Rubber	1.01	0.11
18	Other Manufactured Products	-0.45	0.29

Table 2: Aggregate productivity and productivity dispersion

	Sector	mean ($\log A_{it}$)	sd ($\log A_{it}$)	mean(CV_{it})	sd(CV_{it})
1	Ferric and Non Ferric Metals	11.573	0.289	0.740	0.064
2	Non Metallic Products	10.704	0.208	0.638	0.067
3	Chemical Products	11.480	0.239	0.650	0.096
4	Metallic Products	10.718	0.184	0.603	0.073
5	Agricultural and Industrial Machinery	10.821	0.185	0.606	0.056
6	Office Machinery, Data Processing Machinery	11.230	0.404	0.998	0.442
7	Electrical Material and Electrical Accessories	10.856	0.189	0.644	0.054
8	Vehicles and Motors	10.996	0.282	0.663	0.152
9	Other Transport Material	11.176	0.399	0.765	0.144
10	Meat and Meat Products	11.150	0.138	0.693	0.060
11	Food and Tobacco	10.635	0.137	1.002	0.072
12	Beverages	11.513	0.259	0.579	0.076
13	Textiles and Apparels	10.198	0.168	0.816	0.060
14	Leather products and shoes	10.510	0.198	0.609	0.083
15	Wood and Furniture	10.471	0.183	0.595	0.046
16	Paper, Paper Products and Printing Products	10.806	0.269	0.710	0.049
17	Plastic Products and Rubber	10.885	0.217	0.532	0.048
18	Other Manufactured Products	10.374	0.100	0.822	0.360

Notes: First and third columns report the mean computed over time and second and fourth columns report the standard deviation computed over time.

Table 3: The relationship between aggregate productivity and the dispersion of productivity

	coef (s.e)
Coefficient of variation of productivity (in logs)	-0.364***
	(0.018)
Coefficient of variation of productivity (in logs) $\times$ Dummy $\theta > 1$	0.521***
	(0.025)
Average productivity (in logs)	1.010***
	(0.020)
Test - $\beta_1 = \beta_2$, F (p-value)	3.02 (0.100)
R-squared	0.98
Number of industries	18
Number of observations	270

Notes: Dependent variable: log of aggregate labor productivity. Robust standard errors in brackets.

Table 4: Robustness check

	(1)	(2)	(3)	(4)	(5)
Coefficient of variation of productivity (in logs)	-0.284***	-0.344***	-0.364***	-0.225***	-0.445***
	(0.039)	(0.044)	(0.043)	(0.037)	(0.078)
Coefficient of variation of productivity (in logs) $\times$ Dummy $\theta > 1$	0.547***	0.494***	0.522***	0.215*	1.130***
	(0.046)	(0.092)	(0.099)	(0.112)	(0.151)
Average productivity (in logs)	0.976***	0.970***	1.001***	0.994***	1.033***
	(0.018)	(0.021)	(0.023)	(0.029)	(0.038)
Average productivity (in logs) $\times$ Dummy $\theta > 1$	-	0.057*	-	-	-
		(0.028)			
Capital labor ratio	-	-	0.019	-	-
			(0.028)		
Test - $\beta_1 = \beta_2$, F (p-value)	84.18 (0.000)	3.59 (0.075)	3.08 (0.097)	0.01 (0.924)	27.0 (0.000)
R-squared	0.72	0.98	0.98	0.97	0.96
Number of industries	10	18	18	18	15
Number of observations	150	270	270	270	225

Notes: Dependent variable: log of aggregate labor productivity.

Table 5: Robustness check: The relationship between aggregate productivity and the dispersion of productivity using the level of θ

	coef (s.e)
Coefficient of variation of productivity (in logs)	-0.383***
	(0.044)
Coefficient of variation of productivity (in logs) $\times\theta$	0.242***
	(0.024)
Average productivity (in logs)	0.978***
	(0.026)
R-squared	0.84
Number of industries	18
Number of observations	270

Notes: Dependent variable: log of aggregate labor productivity. Robust standard errors in brackets.